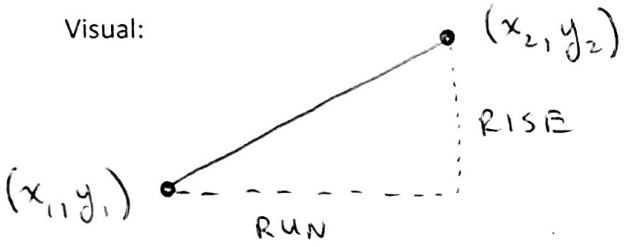
Review: Analytic Geometry | MPM2D

Today we are going to review the following topics:

- Finding the slope between two points using the slope formula
- The relationship between the slopes of perpendicular lines
- Finding the equation of a line algebraically

The Slope of a Line Segment Formula

Visual:



Formula Derivation:

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

You try it: Find the slope of the line segment joining the following pairs of points. Simplify as much as possible.

a) $(2, 3)$ and $(17, 8)$ (x_1, y_1 x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{8 - 3}{17 - 2}$$

$$= 5/15$$

$$= 1/3$$

b) $(-2, 8)$ and $(4, -2)$ (x_1, y_1 x_2, y_2)

$$m = \frac{-2 - 8}{4 - (-2)}$$

$$= \frac{-10}{6}$$

$$= -5/3$$

c) $(-5, 0.5)$ and $(2.5, 0)$ (x_1, y_1 x_2, y_2)

$$m = \frac{0 - 0.5}{2.5 - (-5)}$$

$$= \frac{-0.5 \times 2}{7.5 \times 2}$$

$$= \frac{-1}{15}$$

Parallel and Perpendicular Slopes

Recall that parallel lines have..... the same slope

While the slopes of perpendicular lines are..... negative reciprocals ex/ $\frac{4}{5}$ & $-\frac{5}{4}$

-3 & $\frac{1}{3}$

Complete the following table to reinforce these ideas:

Equation of Line	Slope of Line	Slope of a Parallel Line	Slope of a Perpendicular Line
$y = \frac{2}{3}x + 5$	$2/3$	$2/3$	$-3/2$
$y = -2x + 12$	-2	$-2/1$	$1/2$
$y = 7$	0	$0/1$	undefined

Finding the Equation of a Line Algebraically

We will be doing a lot of work with lines over the next few weeks. An important skill for us to have will be to take some general information about a line, and determine its exact equation. For the following examples, we will draw a visual of each situation. In practice, you can always make a rough sketch if you need to visualize a problem.

We will always use the same strategy:

- Determine the slope of the desired line (if not given)
- Use the slope and a point to find the y-intercept of the desired line

Example 1: Find the equation of the line with a slope of 1/3, through the point (6, 1).

Solution:

$y = mx + b$

$y = \frac{1}{3}x + b$ (plug in slope)

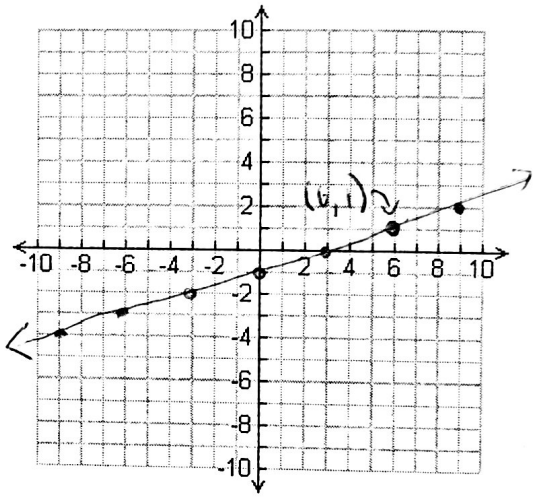
$1 = \frac{1}{3}(6) + b$ (plug in point)

$1 = 2 + b$

$\boxed{b = -1}$

$\boxed{y = \frac{1}{3}x - 1}$

Visual:



$\nearrow 3y = -2x - 9 \rightarrow y = \frac{-2}{3}x - 3$

Example 2: Find the equation of the line that is parallel to $2x + 3y = -9$ through the point $(5, -3)$.

Solution:

$y = mx + b$

$y = \frac{-2}{3}x + b$

$-3 = \frac{-2}{3}(5) + b$

$-3 = \frac{-10}{3} + b$

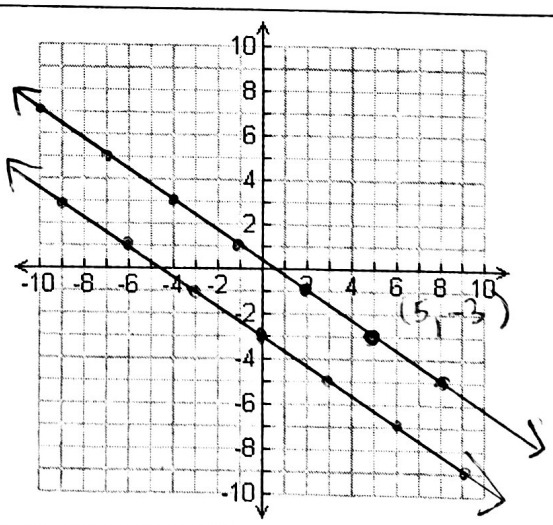
$b = -3 + \frac{10}{3}$

$b = \frac{-9}{3} + \frac{10}{3}$

$\boxed{b = \frac{1}{3}}$

$\boxed{y = \frac{-2}{3}x + \frac{1}{3}}$

Visual:



$x_1\ y_1$ $x_2\ y_2$

Example 3: Find the equation of the line that passes through the points $A(-6,-7)$ and $B(9,3)$

Solution:

$$m = \frac{3-(-7)}{9-(-6)} = \frac{10}{15} = \frac{2}{3}$$

$y = mx + b$

$y = \frac{2}{3}x + b$

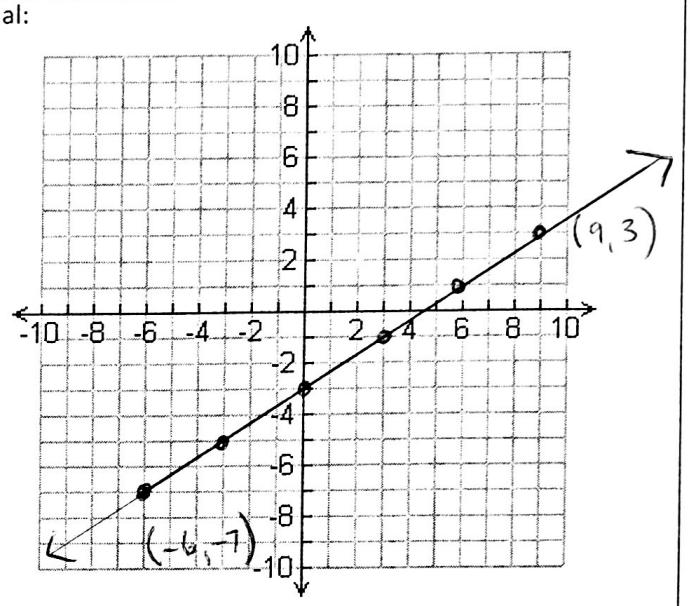
$3 = \frac{2}{3}(9) + b$

$3 = 6 + b$

$b = -3$

$y = \frac{2}{3}x - 3$

Visual:



$-4y = -x + 7$

$y = \frac{1}{4}x - \frac{7}{4}$

Example 4: Find the equation of the line through $(1,1)$ that is perpendicular to $x - 4y = 7$.

Solution:

$m_\perp = -4/1 = -4$

$y = mx + b$

$y = -4x + b$

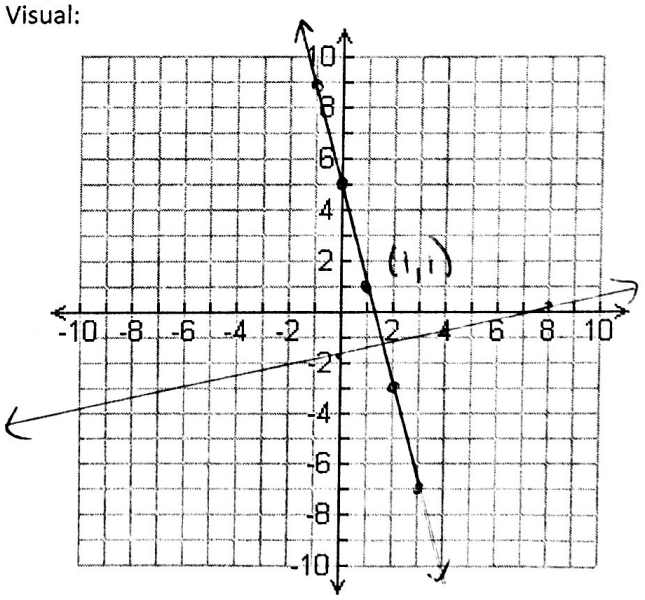
$1 = -4(1) + b$

$1 = -4 + b$

$b = 5$

$y = -4x + 5$

Visual:



SOLUTIONS.

Finding Equations of Lines Practice | MPM2D

1) Find the equation of the line with a slope of $-3/4$, through the point $(8, 1)$.

Solution:

$y = mx + b$

$y = \frac{-3}{4}x + b$

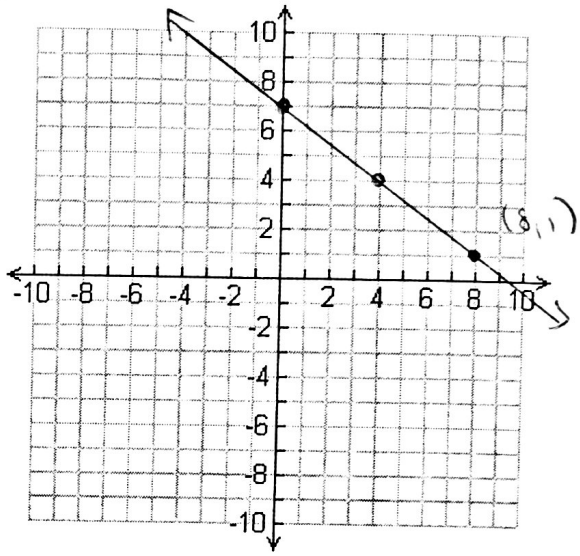
$1 = \frac{-3}{4}(8) + b$

$1 = -6 + b$

$\boxed{b = 7}$

$\boxed{y = \frac{-3}{4}x + 7}$

Visual:



$\nearrow -3y = -4x - 9 \rightarrow y = \frac{4}{3}x + 3$

Example 2: Find the equation of the line that is perpendicular to $4x - 3y = -9$ through the point $(4, 4)$.

Solution: $m_{\perp} = \frac{-3}{4}$

$y = \frac{-3}{4}x + b$

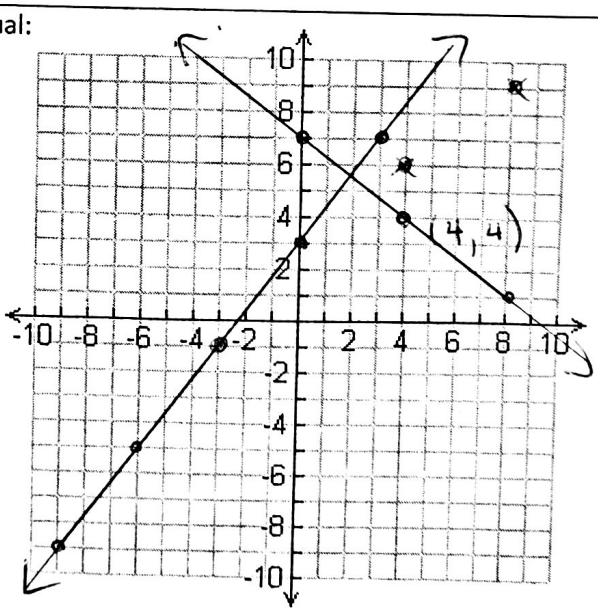
$4 = \frac{-3}{4}(4) + b$

$4 = -3 + b$

$\boxed{b = 7}$

$\boxed{y = \frac{-3}{4}x + 7}$

Visual:



Finding Equations of Lines Practice | MPM2D

3) Find the equation of the line that passes through the points $A(5,-7)$ and $B(1,3)$

$x_1\ y_1 \qquad x_2\ y_2$

Solution:

$$m = \frac{3-(-7)}{1-5} = \frac{10}{-4} = -\frac{5}{2}$$

$$y = \frac{-5}{2}x + b$$

$$3 = \frac{-5}{2}(1) + b$$

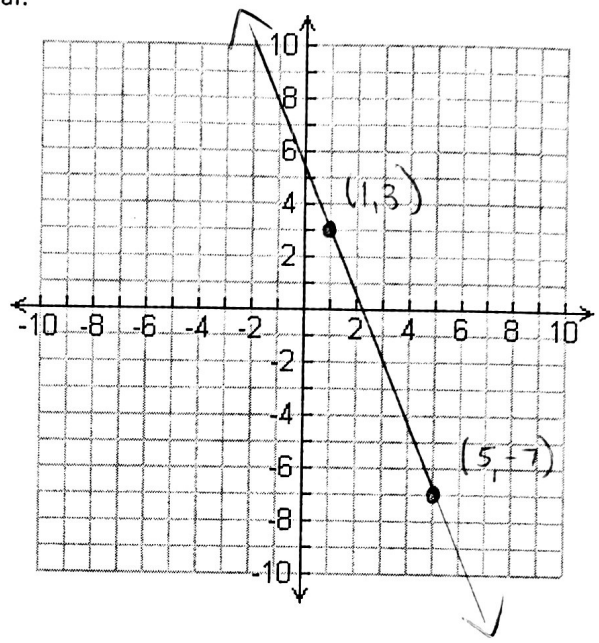
$$3 = \frac{-5}{2} + b$$

$$b = 3 + \frac{5}{2} = \frac{6}{2} + \frac{5}{2}$$

$$\boxed{b = \frac{11}{2}}$$

$$\boxed{y = \frac{-5}{2}x + \frac{11}{2}}$$

Visual:



4) Find the equation of the line with the same y-intercept as $2x + 5y = 10$ that is also perpendicular to $7x - 9y = 5$.

Solution:

$$2x + 5y = 10 \rightarrow 5y = -2x + 10$$

$$y = \frac{-2}{5}x + 2$$

$$(b = 2)$$

$$7x - 9y = 5 \rightarrow -9y = -7x + 5$$

$$y = \frac{7}{9}x - \frac{5}{9}$$

$$(m_{\perp} = -\frac{9}{7})$$

$$\boxed{y = -\frac{9}{7}x + 2}$$

Visual: