

1) Find the midpoint of the following line segments: ② each.

a) $A(1.2, -3.4)$ and $B(-9.1, -7.3)$.

$$M = \left(\frac{1.2 - 9.1}{2}, \frac{-3.4 - 7.3}{2} \right)$$

$$= (-3.95, -5.35)$$

b) $C(102, 12)$ and $D(-48, 216)$.

$$M = \left(\frac{102 - 48}{2}, \frac{12 + 216}{2} \right)$$

$$= (27, 114)$$

2) Find the length of the following line segments to two decimal places: ② each.

a) $A(1.2, -3.4)$ and $B(-9.1, -7.3)$.

$$d = \sqrt{(-9.1 - 1.2)^2 + (-7.3 - (-3.4))^2}$$

$$= \sqrt{(-10.3)^2 + (-3.9)^2}$$

$$= \sqrt{121.3}$$

$$= 11.01$$

b) $C(102, 12)$ and $D(-48, 216)$.

$$d = \sqrt{(-48 - 102)^2 + (216 - 12)^2}$$

$$= \sqrt{(-150)^2 + (204)^2}$$

$$= \sqrt{64,116}$$

$$\approx 253.21$$

3) Consider the triangle formed by the following three points: $A(-7, -2)$, $B(1, 5)$, and $C(5, -7)$.

a) Draw the median from vertex A ②

b) Draw the right bisector of side BC ②

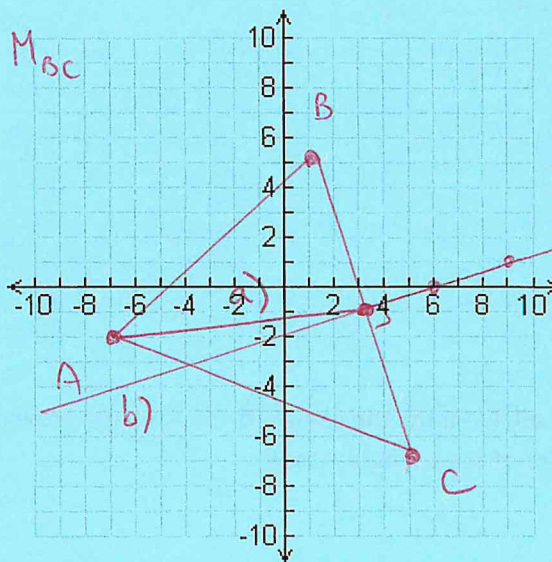
} Need M_{BC}

Space for midpoint/slope calculations:

$$M_{BC} = \left(\frac{1 + 5}{2}, \frac{5 - 7}{2} \right) = (3, -1)$$

$$m_{BC} = \frac{-7 - 5}{5 - 1} = \frac{-12}{4} = -3$$

$$m_{\perp} = \frac{1}{3}$$



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4) [6 marks] Recall an old investigation where found the circumcentre of the triangle formed by the points:

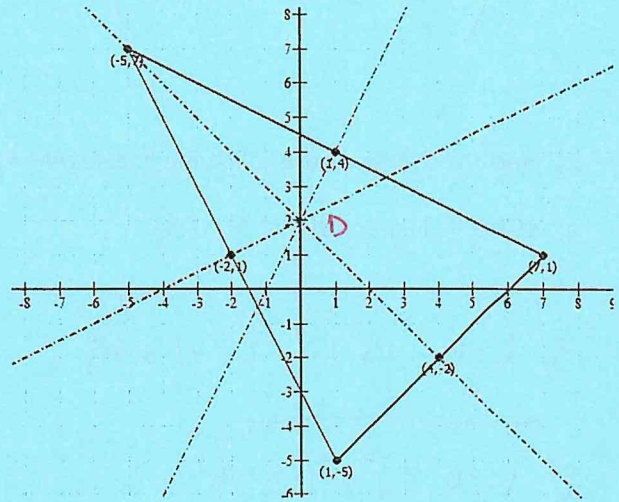
A(1, -5), B(7, 1), and C(-5, 7).

We found the circumcentre (where the 3 right bisectors met) to be at D(0, 2). In this question you will prove that the circumcentre (0, 2) is **equidistant** from all 3 corners of the triangle.

In the space below calculate the **exact distance** of the following 3 line segments: AD, BD, and CD

a) Length of AD

$$\begin{aligned} &= \sqrt{(1-0)^2 + (-5-2)^2} \\ &= \sqrt{1^2 + (-7)^2} \\ &= \sqrt{50} \end{aligned}$$



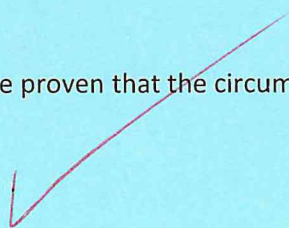
b) Length of BD

$$\begin{aligned} &= \sqrt{(7-0)^2 + (1-2)^2} \\ &= \sqrt{7^2 + (-1)^2} \\ &= \sqrt{50} \end{aligned}$$

c) Length of CD

$$\begin{aligned} &= \sqrt{(-5-0)^2 + (7-2)^2} \\ &= \sqrt{(-5)^2 + 5^2} \\ &= \sqrt{50} \end{aligned}$$

Were all 3 lengths the same? If so, you have proven that the circumcentre is equidistant from the 3 corners of the triangle. Cool!



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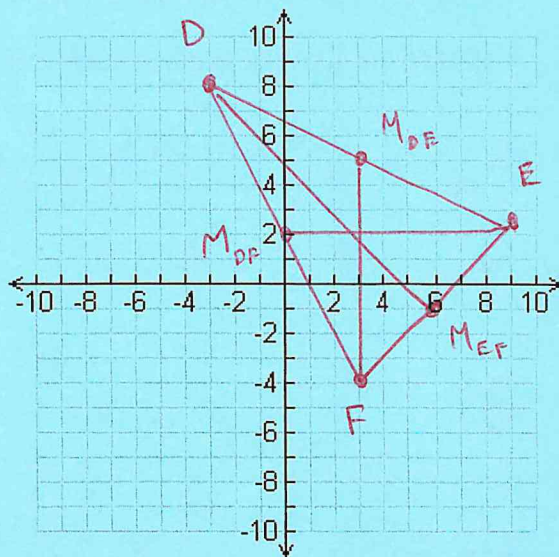
5) Consider the triangle formed by the points D(-3, 8), E(9, 2), and F(3, -4).

a) Draw all 3 medians of this triangle. ©

$$M_{DE} = \left(\frac{-3+9}{2}, \frac{8+2}{2} \right) = (3, 5)$$

$$M_{DF} = \left(\frac{-3+3}{2}, \frac{8+(-4)}{2} \right) = (0, 2)$$

$$M_{EF} = \left(\frac{9+3}{2}, \frac{2+(-4)}{2} \right) = (6, -1)$$



b) Determine the centroid of this triangle using the formula: ②

$$\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

$$= \left(\frac{-3+9+3}{3}, \frac{8+2+(-4)}{3} \right)$$

$$= \left(\frac{9}{3}, \frac{6}{3} \right)$$

$$= (3, 2) \quad \checkmark$$

Analytic Geometry Assignment | MPM2D

5) Find the circumcenter of the triangle formed by the points A(0, 0), B(2, 6), and C(8,0). Use the prompts that Mr. Smith gives you, and always refer to your diagram to check answers! Use extra paper if you feel you need more space to work. ⑩

a) Find the equation of the right bisector of AB.

$$M_{AB} = \left(\frac{0+2}{2}, \frac{0+6}{2} \right) = (1, 3)$$

$$m_{AB} = \frac{6-0}{2-0} = \frac{6}{2} = 3 \quad m_{\perp} = -\frac{1}{3}$$

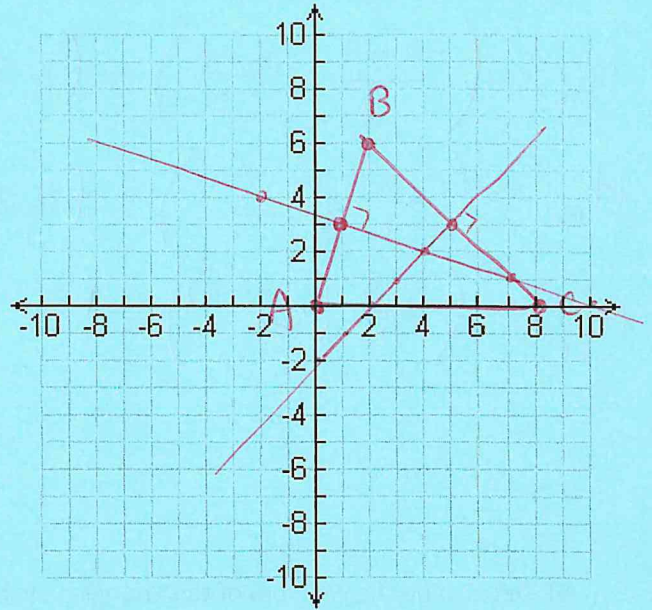
$$y = -\frac{1}{3}x + b$$

$$3 = -\frac{1}{3}(1) + b$$

$$3 = -\frac{1}{3} + b$$

$$b = \frac{10}{3}$$

$$y = -\frac{1}{3}x + \frac{10}{3}$$



b) Find the equation of the right bisector of BC

$$M_{BC} = \left(\frac{2+8}{2}, \frac{6+0}{2} \right) = (5, 3)$$

$$m_{BC} = \frac{0-6}{8-2} = \frac{-6}{6} = -1 \quad m_{\perp} = 1$$

$$y = x + b$$

$$3 = 5 + b$$

$$b = -2$$

$$y = x - 2$$

c) Find the intersection point of your lines you found in a) and b). This is the circumcentre!

$$-\frac{1}{3}x + \frac{10}{3} = x - 2$$

$$-x + 10 = 3x - 6$$

$$-4x = -16$$

$$x = 4$$

$$y = 4 - 2$$

$$y = 2$$

Circumcentre is (4, 2).

SOLUTIONS

Analytic Geometry Mini-Assignment #3 | MPM2D

We have seen that the right bisectors of any two chords of a circle, will intersect at the centre of the circle. You can use this fact to find a point that is **equidistant** (the same distance) from any three points.

1) Mr. Smith is meeting 2 friends for dinner. He has overlaid a Cartesian Plane onto Guelph and each unit represents 1km. Mr. Smith lives at $S(-6, 2)$. His friends, Arnie and Bort live across town at $A(2, 6)$ and $B(3, -1)$ respectively. Mr. Smith wants to meet at a point that is the same distance from everyone's home. You will find this point by:

- Finding the equation of the right bisector of SA
- Finding the equation of the right bisector of SB
- Finding the intersection point (C) of these two right bisectors

Use the space below. Your work should be well organized and neat for full marks, so do your rough work on a separate piece of paper if you need to. Use the provided grid to diagram your solution.

Right Bisector of SA

$$M_{SA} = \left(\frac{2+(-6)}{2}, \frac{6+2}{2} \right) = (-2, 4)$$

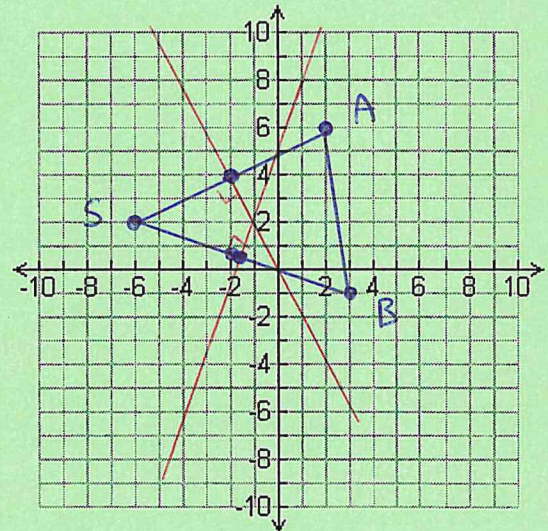
$$m_{SA} = \frac{4}{8} = \frac{1}{2} \quad m_{\perp} = -2$$

$$y = -2x + b$$

$$4 = -2(-2) + b$$

$$4 = 4 + b$$

$$b = 0$$
$$\boxed{y = -2x}$$



Right Bisector of SB

$$M_{SB} = \left(\frac{-6+3}{2}, \frac{2+(-1)}{2} \right) = (-1.5, 0.5)$$

$$m_{SB} = \frac{-3}{9} = -\frac{1}{3} \quad m_{\perp} = 3$$

$$y = 3x + b$$

$$0.5 = 3(-1.5) + b$$

$$0.5 = -4.5 + b$$

$$b = 5$$

$$\boxed{y = 3x + 5}$$

POI: Set (1) = (2)

$$-2x = 3x + 5$$

$$-5x = 5$$

$$\boxed{x = -1}$$

$$y = -2(-1)$$

$$\boxed{y = 2}$$

The right bisectors meet at $(-1, 2)$!

Analytic Geometry Mini-Assignment #3 | MPM2D

2) Check your answer by finding the distance from the point you found to each friend's home:

i) Length of SC

$$(-6, 2) \text{ \& } (-1, 2)$$

$$\begin{aligned} d &= \sqrt{(-1 - (-6))^2 + (2 - 2)^2} \\ &= \sqrt{(5)^2 + 0^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

ii) Length of AC

$$(2, 6) \text{ \& } (-1, 2)$$

$$\begin{aligned} d &= \sqrt{(2 - 6)^2 + (2 - (-1))^2} \\ &= \sqrt{(-4)^2 + 3^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

iii) Length of BC

$$(3, -1) \text{ \& } (-1, 2)$$

$$\begin{aligned} d &= \sqrt{(2 - (-1))^2 + (-1 - 3)^2} \\ &= \sqrt{3^2 + (-4)^2} \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$(-1, 2)$ is where they should meet.
It is 5 km from each house!

Level R	Level 1	Level 2	Level 3	Level 4
Task is incomplete. Work is very unorganized and hard to follow. Mr. Smith may ask you to redo it.	Calculations have major errors. Work is somewhat organized. Instructions were not followed well. Major notation issues.	Calculations have minor errors. Work is somewhat organized. Minor notation issues. Instructions were followed for the most part.	Calculations have minor errors. Work is organized. Proper notation is used. Instructions were properly followed.	All calculations are correct. Work is well organized and proper notation used. Instructions were followed meticulously.