Completing the Square | MPM2D

How can the owner of a snowboard rental business use mathematics to maximize sales or minimize cost? What dimensions of a rectangular field provide the greatest area? Questions like these are answered by finding the maximum or minimum point of a quadratic relation, which occurs at the vertex.

The process of involves changing the first two terms of a quadratic relation of the form $y = ax^2 + bx + c$ into a perfect square while maintaining the balance of the original relation.

completing the square

- a process for expressing $y = ax^2 + bx + c$ in the form $y = a(x - h)^2 + k$

We will talk about two methods today for getting the vertex form of a quadratic relation, starting from standard form.

- Using the axis of symmetry formula: $x = {-b}/{2a}$
- Completing the square (an algebraic method)

Example: Convert the relation $y = x^2 + 6x + 2$ into vertex form.

$\left(\frac{6}{2}\right)^2 = 9$

Method 1: Using $x = {-b}/{2a}$

AOS: $x = \frac{-6}{2(1)} = -3$

$y = (-3)^2 + 6(-3) + 2$

$= 9 - 18 + 2$

$= -7$

vertex form: $y = (x+3)^2 - 7$

Method 2: By completing the square

$y = \underline{x^2 + 6x} + 2$

$= x^2 + 6x + 9 - 9 + 2$

$= (x+3)^2 - 7$

✓

You try it: Convert the relation $y = x^2 - 10x - 7$ into vertex form.

$\left(\frac{-10}{2}\right)^2 = 25$

Method 1: Using $x = {-b}/{2a}$

AOS: $x = \frac{10}{2(1)} = 5$

$y = 5^2 - 10(5) - 7$

$= 25 - 50 - 7$

$= -32$

vertex form: $y = (x-5)^2 - 32$

Method 2: By completing the square

$y = x^2 - 10x - 7$

$= x^2 - 10x + 25 - 25 - 7$

$= (x-5)^2 - 32$

In the previous two examples, the value of "a" was 1, making it somewhat straightforward to complete the square. Now we will do some examples where the "a" value is not 1.

Example: Convert the relation $y = 2x^2 - 16x + 5$ into vertex form.

$\left(\frac{-8}{2}\right)^2 = 16$

Method 1: Using $x = {}^{-b}/_{2a}$

AOS: $x = \frac{16}{2(2)} = 4$

$y = 2(4)^2 - 16(4) + 5$

$= 32 - 64 + 5$

$= -27$

$y = 2(x-4)^2 - 27$

Method 2: By completing the square

Relation:	$y = 2x^2 - 16x + 5$
Factor the "a" value from the first two terms	$= 2(x^2 - 8x) + 5$
Complete the square	$= 2(x^2 - 8x + 16 - 16) + 5$
Distribute the extra term	$= 2(x^2 - 8x + 16) - 32 + 5$
Simplify	$= 2(x-4)^2 - 27$

Note: If using method 1, do not forget to carry down the correct "a" value.

Let's try two more using method 2. The second will have decimal values, that's ok.

$\left(\frac{+4}{2}\right)^2 = 4$

Relation:	$y = -3x^2 - 12x + 1$
Factor the "a" value from the first two terms	$= -3(x^2 + 4x) + 1$
Complete the square	$= -3(x^2 + 4x + 4 - 4) + 1$
Distribute the extra term	$= -3(x^2 + 4x + 4) + 12 + 1$
Simplify	$= -3(x+2)^2 + 13$

$\left(\frac{-3.5}{2}\right)^2 =$

Relation:	$y = 2x^2 - 7x + 1.5$
Factor the "a" value from the first two terms	$= 2(x^2 - 3.5x) + 1.5$
Complete the square	$= 2(x^2 - 3.5x + 3.0625 - 3.0625) + 1.5$
Distribute the extra term	$= 2(x^2 - 3.5x + 3.0625) - 6.125 + 1.5$
Simplify	$= 2(x - 1.75)^2 - 4.625$

On Monday, we will do some problem solving using this skill.