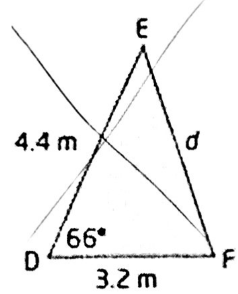
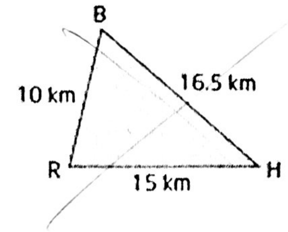
Which of the following triangles can be solved by using the Sine Law?

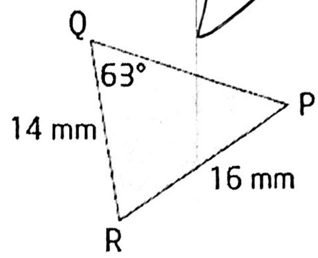
a)



b)



c)



KEY IDEA: Triangles with 2 sides and a contained angle (SAS) cannot be solved with the Sine Law. Same with triangles with all 3 sides (SSS).

Investigation: Mr. Smith will throw up a few triangles in a geometry program. Make the indicated calculations, and see if you can notice a relationship.

Triangle #1			
Side a	Side b	Side c	Angle A
6.15	7.98	7.28	47.27°
$a^2 =$ 37.82	$b^2 + c^2 =$ 116.68	$2bc \cdot cosA =$	2(7.98)(7.28) cos 47.27° 78.84

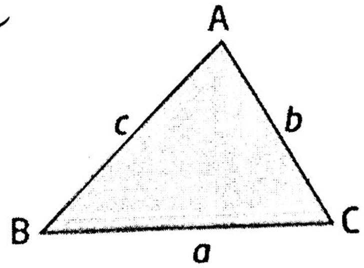
Triangle #2			
Side a	Side b	Side c	Angle A
7.28	12.62	7.28	29.98°
$a^2 =$ 53	$b^2 + c^2 =$ 212.26	$2bc \cdot cosA =$	2(12.62)(7.28) cos 29.98° 159.16

Triangle #3			
Side a	Side b	Side c	Angle A
8.63	4.6	7.28	90.22
$a^2 =$ 74.48	$b^2 + c^2 =$ 74.16	$2bc \cdot cosA =$ −0.26	

Did you notice a relationship between a^2, $b^2 + c^2$, and $2bc \cdot cosA$?

It looks like $a^2 = b^2 + c^2 - 2bc \cdot \cos A$

Summary: The Cosine Law For any ΔABC



$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

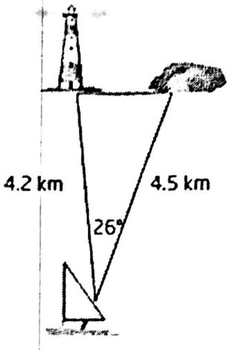
$c^2 = a^2 + b^2 - 2ab \cdot \cos C$

Notes:

- The Cosine Law can be used to find sides AND angles.
- If you have a right angle, the Cosine Law is equivalent to the Pythagorean Theorem.
- We use the Cosine Law in the following situations: SAS, SSS

Finding sides with the Cosine Law

Example: A boat is sailing north through a narrow strait. Through one particularly narrow section, a lighthouse marks the western shoreline, while a buoy indicates a rock hazard directly east of the lighthouse, as shown. What is the width of this section of the strait, to the nearest tenth of a kilometre?



$c^2 = a^2 + b^2 - 2ab \cdot \cos C$

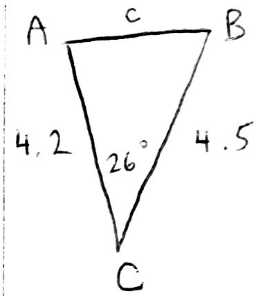
$= \underline{4.5^2} + \underline{4.2^2} - \underline{2(4.5)(4.2)\cos 26^\circ}$

$= 20.25 + 17.64 - 33.97$

$= 3.92$

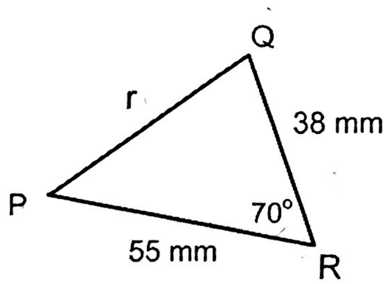
$c = \sqrt{3.92}$

$c \doteq 1.98$ km or about 2 km.



You try it:

a) Find side "r" in this triangle



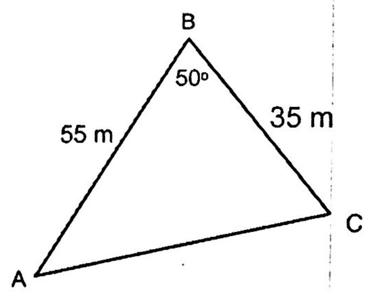
$r^2 = p^2 + q^2 - 2pq \cdot \cos R$

$= 38^2 + 55^2 - 2(38)(55)\cos 70$

$= 3,039.36$

$\boxed{r = 55.1 \text{ mm}}$

b) Find side "b" in this triangle



$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

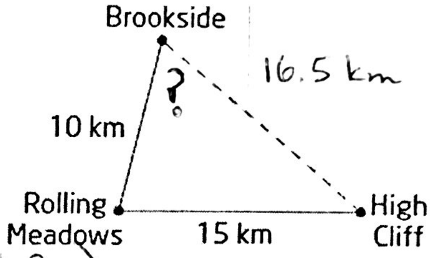
$= 35^2 + 55^2 - 2(35)(55)\cos 50^\circ$

$= 1775.27$

$\boxed{b \doteq 42.1 \text{ m}}$

The Cosine Law: Finding Angles | MPM2D

Example: Three towns are connected by two roads, as shown. A third road is planned that will directly connect Brookside and High Cliff, which are 16.5 km apart. Find the angle, to the nearest tenth of a degree, between the new road and the existing road from Rolling Meadows to Brookside.

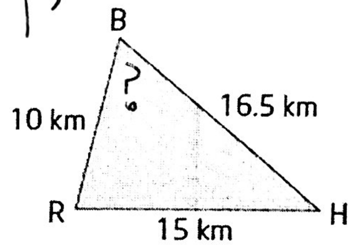


Method 1: Substitute, then rearrange...

$b^2 = r^2 + h^2 - 2rh\cos B$

$15^2 = 16.5^2 + 10^2 - 2(16.5)(10)\cos B$

$225 = 272.25 + 100 - 330\cos B$ (simplify)



$225 - 272.25 - 100 = -330\cos B$

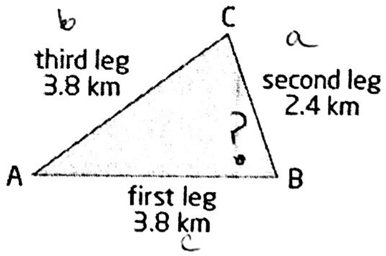
$\dfrac{-147.25}{-330} = \dfrac{-330\cos B}{-330}$

$0.4462 = \cos B$

$B = \cos^{-1}(0.4462) \doteq 63.5^\circ$

Example: Runners in a 10-K race follow a triangular course. The three legs of the race, in order, are 3.8 km, 2.4 km, and 3.8 km. Find the angle between the first and second leg.

Method 2: Rearrange, then substitute...



$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

$\dfrac{b^2 - a^2 - c^2}{-2ac} = \dfrac{-2ac \cdot \cos B}{-2ac}$

$\dfrac{b^2 - a^2 - c^2}{-2ac} = \cos B$

$\boxed{\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}}$

$\cos B = \dfrac{2.4^2 + 3.8^2 - 3.8^2}{2(2.4)(3.8)}$

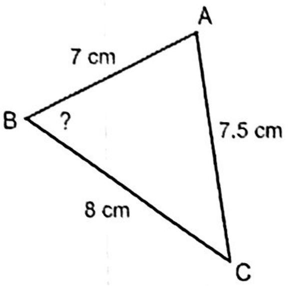
$= \dfrac{5.76}{18.24}$

$= 0.3158$

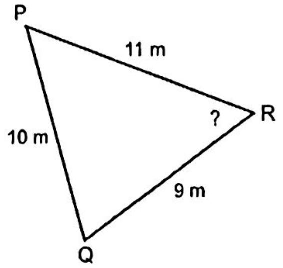
$\boxed{B = 71.6^\circ}$

Try setting up the Cosine Law formula (just the formula) to solve for the indicated angle.

a)



b)



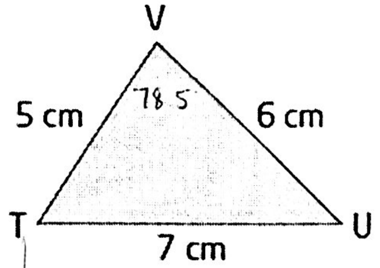
$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

~~cosB~~ =

cosR = $\dfrac{p^2 + q^2 - r^2}{2pq}$

Example: Solve the following triangle completely…

1) Find V (using the Cosine Law)
2) Find T (using the Sine Law)
3) Find U (angles add to 180°)



1) $\cos V = \dfrac{t^2 + u^2 - v^2}{2tu}$

$= \dfrac{6^2 + 5^2 - 7^2}{2(6)(5)}$

$= \dfrac{12}{60}$

$= 0.2$

$\boxed{V = 78.5°}$

2) $\dfrac{\sin T}{6} = \dfrac{\sin 78.5}{7}$

$\sin T = \dfrac{6 \sin 78.5}{7}$

$\sin T = 0.8399$

$\boxed{T = 57.1°}$

3) $180 - 57.1 - 78.5$

$\boxed{U = 44.4°}$

Angle T = 57.1°	*Angle U =* 44.4°	*Angle V =* 78.5°