

Difference of Squares Factoring | MPM2D

Motivation: Expand the following binomial products...

$$\begin{aligned} \text{a) } & (x-4)(x+4) \\ & = x^2 + 4x - 4x - 16 \\ & = x^2 - 16 \end{aligned}$$

$$\begin{aligned} \text{b) } & (x+6)(x-6) \\ & = x^2 - 6x + 6x - 36 \\ & = x^2 - 36 \end{aligned}$$

$$\begin{aligned} \text{c) } & (2x-3)(2x+3) \\ & = 4x^2 + 6x - 6x - 9 \\ & = 4x^2 - 9 \end{aligned}$$

$$\begin{aligned} \text{d) } & (3x-2y)(3x+2y) \\ & = 9x^2 - 4y^2 \end{aligned}$$

$$\begin{aligned} \text{e) } & (4a^2-3b^2)(4a^2+3b^2) \\ & = 16a^4 - 9b^4 \end{aligned}$$

Recall that multiplying a binomial by its conjugate produced a **difference of squares**:

$$(a+b)(a-b) = a^2 - b^2$$

KEY IDEA: We can work in reverse to factor a difference of squares $a^2 - b^2$ into $(a+b)(a-b)$.

Example: Verify that each of the following are **difference of squares** expressions. Then, factor them.

$$\begin{aligned} \text{a) } & x^2 - 144 \\ & = (x+12)(x-12) \end{aligned}$$

$$\begin{aligned} \text{b) } & 25x^2 - 4 \\ & = (5x+2)(5x-2) \end{aligned}$$

$$\begin{aligned} \text{c) } & 25x^2 - 36 \\ & = (5x+6)(5x-6) \end{aligned}$$

$$\begin{aligned} \sqrt{x^2} & = x \\ \sqrt{144} & = 12 \end{aligned}$$

$$\begin{aligned} \sqrt{25x^2} & = 5x \\ \sqrt{4} & = 2 \end{aligned}$$

$$\begin{aligned} \text{d) } & x^4 - 9 \\ & = (x^2+3)(x^2-3) \end{aligned}$$

$$\begin{aligned} \text{e) } & 9a^2b^2 - 4c^2 \\ & = (3ab+2c)(3ab-2c) \end{aligned}$$

$$\begin{aligned} \text{f) } & a^4b^6 - 49c^8 \\ & = (a^2b^3+7c^4)(a^2b^3-7c^4) \end{aligned}$$

$$\begin{aligned} \sqrt{x^4} & = x^2 \\ \sqrt{9} & = 3 \end{aligned}$$

$$\begin{aligned} \sqrt{9a^2b^2} & = 3ab \\ \sqrt{4c^2} & = 2c \end{aligned}$$

Difference of Squares Factoring | MPM2D

Don't think of this as one more method of factoring to memorize, think of it as a shortcut to help make your factoring even easier.

Remember that you should always check to common factor first.

Example: Common factor the following expressions, and then apply difference of squares factoring.

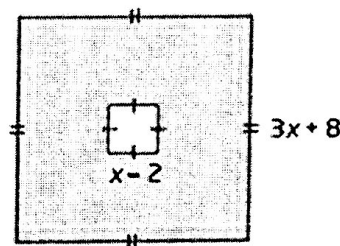
$$\begin{aligned} \text{a) } 5x^2 - 20 \\ &= 5(x^2 - 4) \\ &= 5(x+2)(x-2) \end{aligned}$$

$$\begin{aligned} \text{b) } 7x^2 - 63 \\ &= 7(x^2 - 9) \\ &= 7(x+3)(x-3) \end{aligned}$$

$$\begin{aligned} \text{c) } 25x^2y^2 - 100 \\ &= 25(x^2y^2 - 4) \\ &= 25(xy + 2)(xy - 2) \end{aligned}$$

Example: Modelling an area

a) Determine an expression for the shaded area diagrammed at the right.



$$A = (3x+8)^2 - (x-2)^2$$

b) Factor this expression as a difference of squares

$$\begin{aligned} &= (3x+8 + (x-2))(3x+8 - (x-2)) \\ &= (3x+8 + x-2)(3x+8 - x+2) \\ &= (4x+6)(2x+10) \\ &= 2(2x+3) \cdot 2(x+5) \quad \rightarrow = 4(2x+3)(x+5) \end{aligned}$$

Now it is time to practice:

- Head to page 253 in your text, copy down the first two key concepts. The third we will look at tomorrow.
- 1, 2, 5ad, 6dfi, 7, 9, 10bc