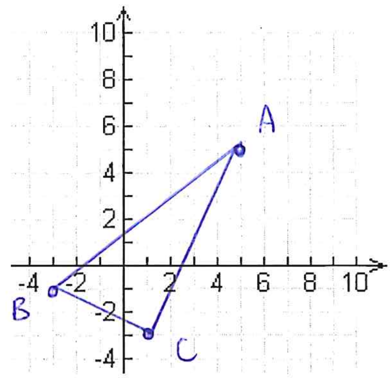
Warmup: Applying Slope, Length, Midpoint

The vertices of ΔABC are A(5, 5), B(-3, -1), and C(1, -3). We are going to determine if ΔABC is a right-angle triangle. In order to prove certain geometric properties, you will eventually have to decide whether calculating slopes, midpoints, or lengths will help you.



To start, determine the lengths and slopes of each side. If you'd like, split up the work with your seat partner.

Side	Length (Exact)	Slope
x_1 y_1 $(5, 5)$ x_2 y_2 $(-3, -1)$ AB	$d = \sqrt{(-3-5)^2 + (-1-5)^2}$ $= \sqrt{(-8)^2 + (-6)^2}$ $= \sqrt{100}$ $= 10$	$m = \frac{-1-5}{-3-5}$ $= \frac{-6}{-8}$ $= 3/4$
x_1 y_1 $(5, 5)$ x_2 y_2 $(1, -3)$ AC	$d = \sqrt{(1-5)^2 + (-3-5)^2}$ $= \sqrt{(-4)^2 + (-8)^2}$ $= \sqrt{80}$	$m = \frac{-3-5}{1-5}$ $= \frac{-8}{-4}$ $= 2$
x_1 y_1 $(-3, -1)$ x_2 y_2 $(1, -3)$ BC	$d = \sqrt{(1-(-3))^2 + (-3-(-1))^2}$ $= \sqrt{4^2 + (-2)^2}$ $= \sqrt{20}$	$m = \frac{-3-(-1)}{1-(-3)}$ $= \frac{-2}{4}$ $= -1/2$

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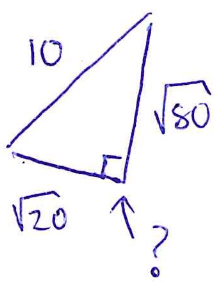
Analysis:

1) How can you use the slopes of each side to determine if ΔABC is a right angle triangle? Is it?

Since AC & BC are perpendicular (slopes are negative reciprocals) ΔABC is a right-angle triangle.

2) How can you use the lengths of each side to determine if ΔABC is a right angle triangle? Is it?



Do the sides satisfy

$c^2 = a^2 + b^2$?

$10^2 = (\sqrt{80})^2 + (\sqrt{20})^2$

$100 = 80 + 20$

$100 = 100$ ✓

The Shortest Distance From a Point to a Line | MPM2D

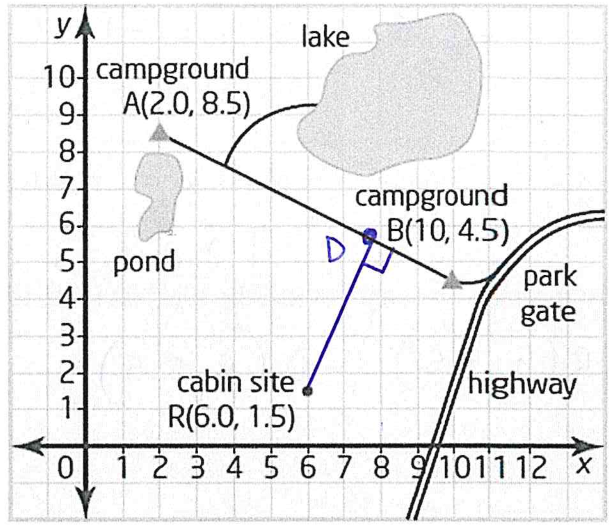
Ideally, the route of a power line should be as short as possible. A shorter route reduces the construction cost as well as the energy losses due to the resistance of the wire. Engineers use analytic geometry to find the best route for the transmission lines that deliver electricity throughout the province. Analytic geometry is also a powerful tool for designing roads, buildings, pipelines, industrial machinery, and consumer products.

A ranger cabin is to be built in a flat wooded area near the straight road that connects the two campgrounds in a park.

A new side road will connect the cabin to the campground road. On the park map, the campgrounds have coordinates A(2.0, 8.5) and B(10.0, 4.5), while the site for the cabin is at R(6.0, 1.5).

Each unit on the map grid represents 500 m.

Task: Find the shortest route from the cabin site to the road (this will minimize the forest being cut down), and calculate the length of this shortest route.



KEY IDEA: The shortest route from a point to a line is perpendicular to the line.

1) Find the slope of the road connecting A(2.0, 8.5) and B(10, 4.5), and the slope of the shortest route RD.

$x_1 \; y_1 \qquad x_2 \; y_2$

$$m_{AB} = \frac{4.5 - 8.5}{10 - 2} = \frac{-4}{8} = \frac{-1}{2} \qquad m_{RD} = 2$$

$$= -0.5$$

2) Find the equation of AB and RD.

i) For AB, use A(2.0, 8.5) (x, y)

$y = -0.5x + b$

$8.5 = -0.5(2) + b$

$8.5 = -1 + b$

$b = 9.5$

$\boxed{y = -0.5x + 9.5}$

ii) For RD, use R(6.0, 1.5) (x, y)

$y = 2x + b$

$1.5 = 2(6) + b$

$1.5 = 12 + b$

$b = -10.5$

$\boxed{y = 2x - 10.5}$

3) Use the method of substitution to find the intersection of AB and RD.

Set ① = ②

$$-0.5x + 9.5 = 2x - 10.5$$
$$-2x \qquad -2x$$
$$-2.5x + 9.5 = -10.5$$
$$-9.5 \qquad -9.5$$
$$-2.5x = -20$$
$$\boxed{x = 8}$$

Sub $x = 8$ in ②

$$y = 2(8) - 10.5$$
$$\boxed{y = 5.5}$$

4) Calculate the length of RD (the shortest route to the campground road) to the nearest meter.

$R(\overset{x_1}{6}, \overset{y_1}{1.5})$ & $D(\overset{x_2}{8}, \overset{y_2}{5.5})$

$$d = \sqrt{(8-6)^2 + (5.5-1.5)^2}$$
$$= \sqrt{2^2 + 4^2}$$
$$= \sqrt{20}$$
$$= 4.47 \text{ units}$$

Conclusion: To build the shortest route possible, connect the roads at (8, 5.5). The road would $4.47 \times 500 = 2{,}235$m.

Summary: To find the shortest distance from a point to a line...

- Find the slope and equation of the given line (if not already given)
- Take the negative reciprocal to get the slope of the shortest route
- Find the equation of the shortest route
- Use substitution to find the point of intersection
- Find the distance of the shortest route

Note: Using a diagram can help you visualize the problem as you go!