

Warmup: Finding Equations of Lines | MPM2D

- 1) Find the equation of the line with a slope of $\frac{5}{4}$, through the point $(-8, -9)$.

Solution:

$$y = mx + b$$

$$y = \frac{5}{4}x + b$$

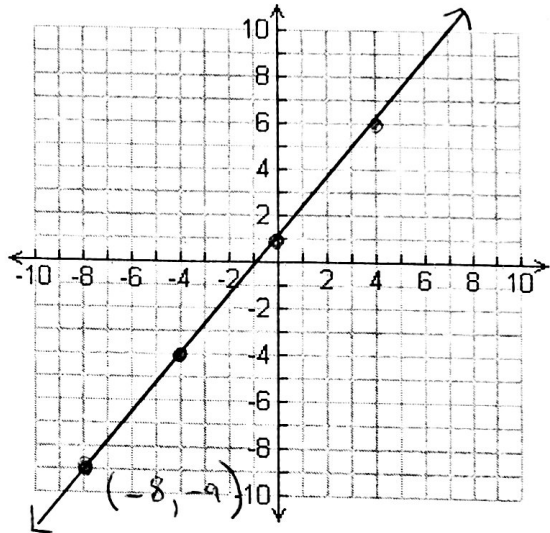
$$-9 = \frac{5}{4}(-8) + b$$

$$-9 = -10 + b$$

$$b = 1$$

$$y = \frac{5}{4}x + 1$$

Visual:



- 2) Find the equation of the line that is perpendicular to $y = \frac{2}{3}x - 5$ through the point $(1, 4)$.

Solution:

$$m_{\perp} = -\frac{3}{2}$$

$$y = -\frac{3}{2}x + b$$

$$4 = -\frac{3}{2}(1) + b$$

$$4 = -\frac{3}{2} + b$$

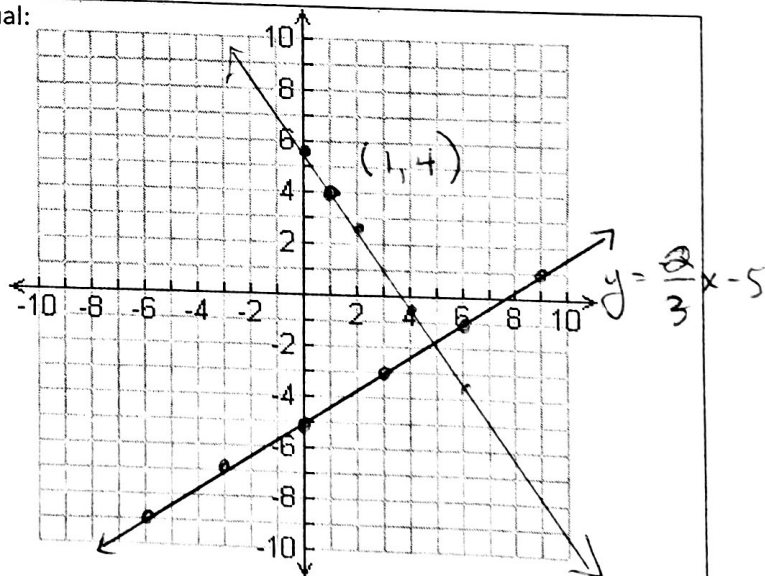
$$4 + \frac{3}{2} = b$$

$$\frac{8}{2} + \frac{3}{2} = b$$

$$b = \frac{11}{2}$$

$$y = -\frac{3}{2}x + \frac{11}{2}$$

Visual:



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3) Find the equation of the line that passes through the points $A(2, -3)$ and $B(-1, -4)$

Solution:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - (-3)}{-1 - 2} = \frac{-1}{-3} = \frac{1}{3}$$

$$y = \frac{1}{3}x + b$$

$$-4 = \frac{1}{3}(-1) + b$$

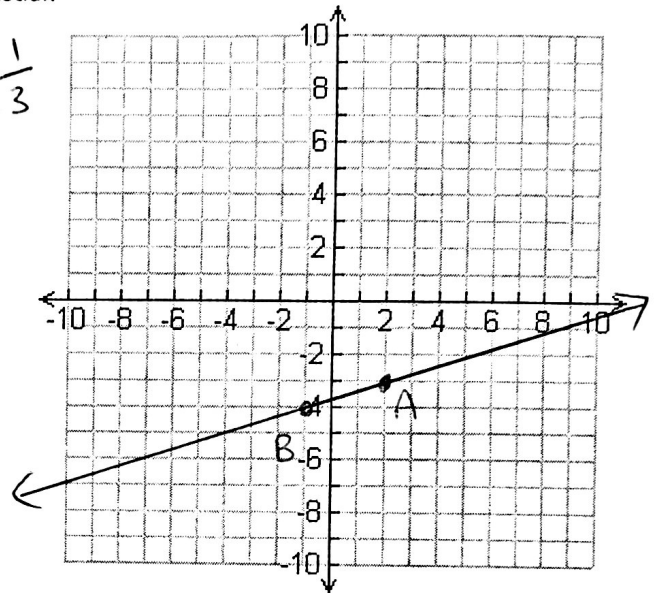
$$-4 = -\frac{1}{3} + b$$

$$-\frac{12}{3} + \frac{1}{3} = b$$

$$-\frac{11}{3} = b$$

$$y = \frac{1}{3}x - \frac{11}{3}$$

Visual:



The Midpoint of a Line Segment | MPM2D

Coordinates on a Cartesian Plane are a convenient way to specify a location. Machinists use coordinates to control machines that shape and machine parts. People that work in construction and design use coordinates in a computer program to simplify their design work and calculations.

Today we want to develop a formula that will find the midpoint of a line segment.

Midpoint: A point that divides a line segment into 2 equal parts.

Example: Using slope to find a midpoint

Determine the rise and run of the line segment joining $A(-2, -3)$ and $B(8, 5)$. Use these to determine the midpoint of this line segment.

$$\text{rise} = y_2 - y_1 = 5 - (-3) = 8$$

$$\text{run} = x_2 - x_1 = 8 - (-2) = 10$$

$$\begin{aligned} \text{Midpoint } M &= \left(-2 + \frac{\text{run}}{2}, -3 + \frac{\text{rise}}{2}\right) \\ &= \left(-2 + \frac{10}{2}, -3 + \frac{8}{2}\right) \\ &= (3, 1) \end{aligned}$$

Example: Using a formula to find a midpoint

Formula:

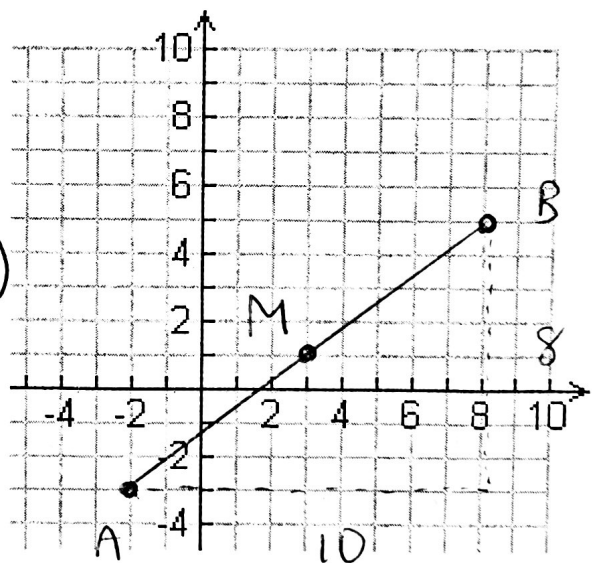
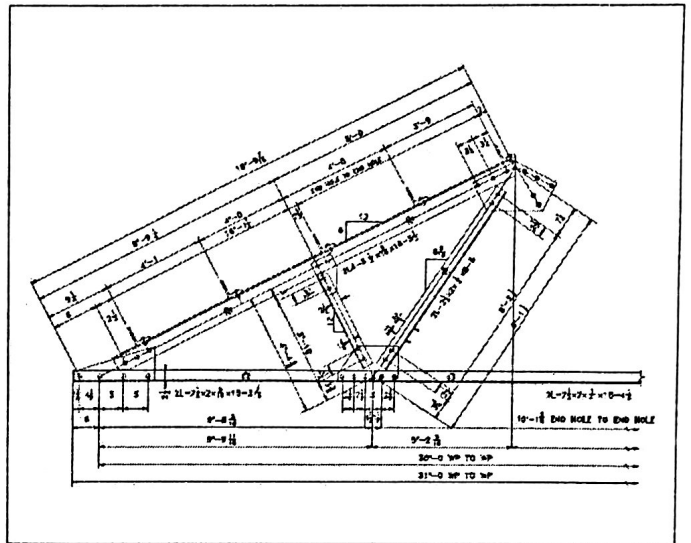
Given 2 points

$A(x_1, y_1)$ & $B(x_2, y_2)$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Sample Calculation:

$$\begin{aligned} M &= \left(\frac{-2 + 8}{2}, \frac{-3 + 5}{2}\right) \\ &= \left(\frac{6}{2}, \frac{2}{2}\right) \\ &= (3, 1) \end{aligned}$$



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You try it: Use the midpoint formula to find the midpoint of the following line segments.

a) $A(1, -3)$ and $B(4, 3)$.

b) $C(12, 33)$ and $D(48, -15)$.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$= \left(\frac{1 + 4}{2}, \frac{-3 + 3}{2} \right)$$

$$= \left(\frac{5}{2}, \frac{0}{2} \right)$$

$$= \left(\frac{5}{2}, 0 \right)$$

$$M = \left(\frac{12 + 48}{2}, \frac{33 + (-15)}{2} \right)$$

$$= \left(\frac{60}{2}, \frac{18}{2} \right)$$

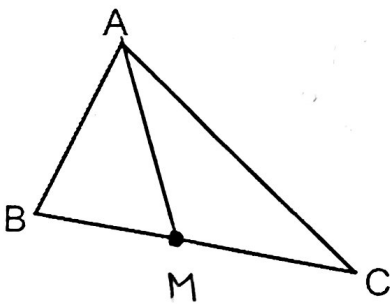
$$= (30, 9)$$

Example 3: Medians of Triangles

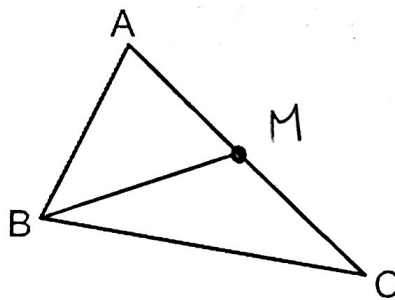
A median of a triangle is... a line from a vertex of the triangle to the midpoint of the opposite side.

Draw the three medians for triangle ABC shown below:

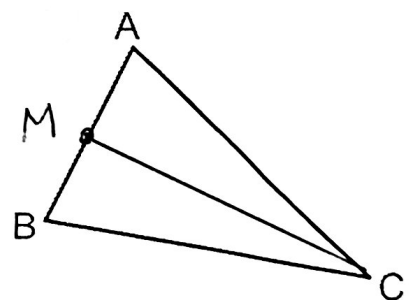
From vertex A:



From vertex B:



From vertex C:



Notes:

- Every triangle has 3 medians
- The 3 medians of triangle always meet at a point.

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Example: Find the equation of all 3 medians in the triangle formed by the points $A(4, 4)$, $B(10, 6)$, and $C(6, 0)$.

Median from A:

$$M_{BC} = \left(\frac{10+6}{2}, \frac{6+0}{2} \right) = (8, 3)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 4}{8 - 4} = -\frac{1}{4}$$

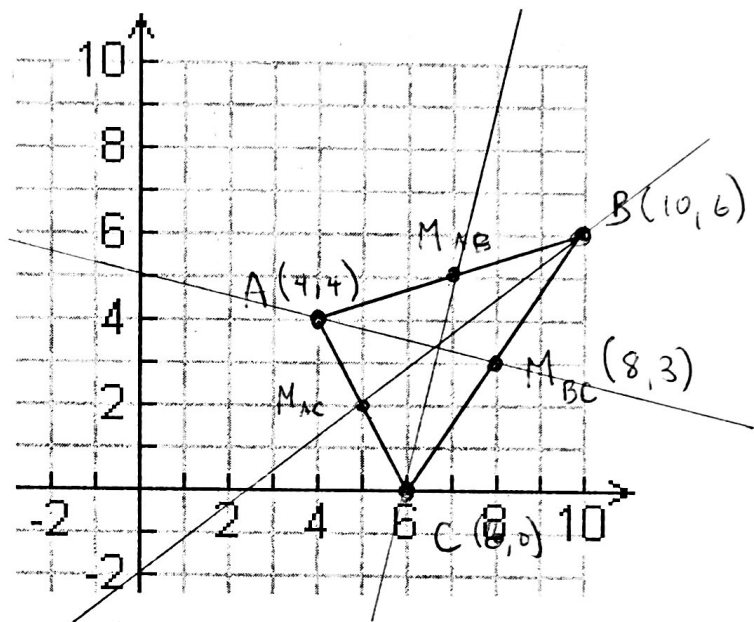
$$y = -\frac{1}{4}x + b$$

$$4 = -\frac{1}{4}(4) + b$$

$$4 = -1 + b$$

$$b = 5$$

$$y = -\frac{1}{4}x + 5$$



You try the following two...

Median from B:

$$M_{AC} = \left(\frac{4+6}{2}, \frac{4+0}{2} \right) = (5, 2)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 4}{5 - 4} = -2$$

$$y = -2x + b$$

$$2 = -2(5) + b$$

$$2 = -10 + b$$

$$b = 12$$

$$y = -2x + 12$$

Median from C:

$(6, 0)$ to $(7, 5)$
 $\rightarrow C$ to M_{AB}

$$M_{AB} = \left(\frac{4+10}{2}, \frac{4+6}{2} \right) = (7, 5)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{7 - 6} = \frac{5}{1} = 5$$

$$y = 5x + b$$

$$0 = 5(6) + b$$

$$0 = 30 + b$$

$$b = -30$$

$$y = 5x - 30$$