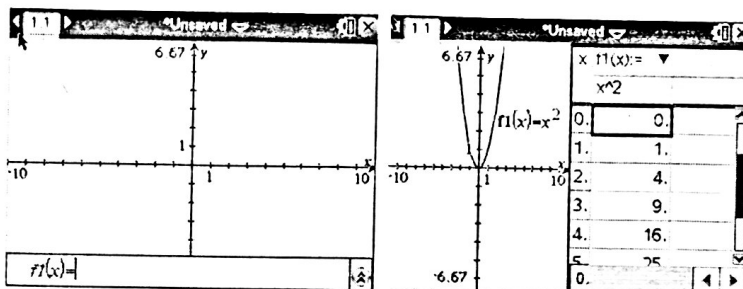


The Factored Form of a Quadratic Relation | MPM2D

In this investigation we will look at a new form of quadratic relations. Our today we will be looking at the x-intercepts or zeros of parabolas.

TECHNOLOGY OPTION

If you are using a TI-nspire, add a new "Graph Document" and enter your relation. You can bring up a table of values by pressing "ctrl" then "T".



Complete the following tables fully, describing the transformations that each graph undergoes from the basic parabola $y = x^2$.

Mr. Smith will do the first one with you:

Relation #1: $y = (x - 2)(x + 2)$		
x	y	Graph:
-3	5	
-2	0	
-1	-3	
0	-4	
1	-3	
2	0	
3	5	
X-intercepts: $(-2, 0)$ & $(2, 0)$		
Step Pattern: $1, 3, 5$		

Reflect: Based on what you saw with this first relation, could you predict the x-intercepts and step pattern of the following relations?

Relation	x-intercepts	Step Pattern
$y = (x + 1)(x + 5)$		
$y = 2(x - 1)(x + 3)$		

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Relation #2: $y = (x + 1)(x + 5)$

x	y	Graph:
-6	5	
-5	0	
-4	-3	
-3	-4	
-2	-3	
-1	0	
0	5	
X-intercepts: $(-1, 0)$ & $(-5, 0)$		
Step Pattern: 1, 3, 5		

Relation #3: $y = 2(x - 1)(x + 3)$

x	y	Graph:
-3	0	
-2	-6	
-1	-8	
0	-6	
1	0	
2	10	
3	24	
X-intercepts: $(-3, 0)$ & $(1, 0)$		
Step Pattern: 2, 6, 10		

How did you do with your predictions? Did you need to make any adjustments? Try predicting the x-intercepts and step pattern for these relations.

Relation	x-intercepts	Step Pattern
$y = 0.5(x - 1)(x + 7)$	$(1, 0)$ & $(-7, 0)$	0.5, 1.5, 2.5
$y = 5(x + 2)(x + 8)$	$(-2, 0)$ & $(-8, 0)$	5, 15, 25

Factored Form of a Quadratic Relation | MFM2P

The factored form of a quadratic relation is given by:

$$y = a(x - s)(x - t)$$

Where: "s" and "t" tells you....

The x-intercepts / zeros. They are $(s, 0)$ & $(t, 0)$

"a" tells you...

The step pattern $1a, 3a, 5a$

Notes: We now have seen 3 different forms of quadratic relations...

1) $y = a(x - s)(x - t)$ 2) $y = a(x - h)^2 + k$

3) $y = ax^2 + bx + c$

All 3 of these forms are quadratic, and produce a parabola if graphed.

Example: Sketch a graph of $y = 2(x - 2)(x - 6)$ and label the x-intercepts, vertex, and axis of symmetry.

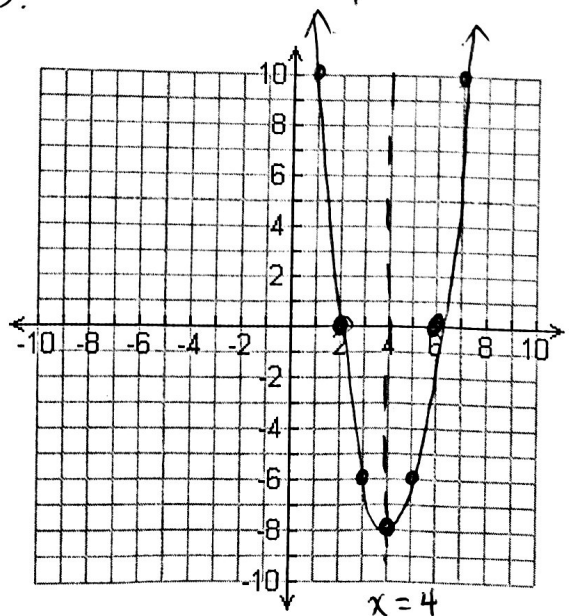
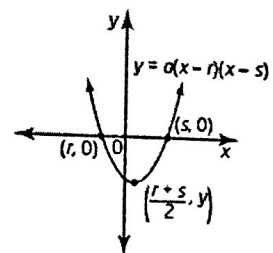
KEY IDEA: The axis of symmetry is halfway between the x-intercepts.

To find the axis of symmetry, average the two x-intercepts.

$$\text{AOS} = \frac{2 + 6}{2} = 4$$

$$\begin{aligned} \text{Vertex: } y &= 2(4 - 2)(4 - 6) \\ &= 2(2)(-2) \\ &= -8 \end{aligned}$$

$(4, -8)$ is the vertex



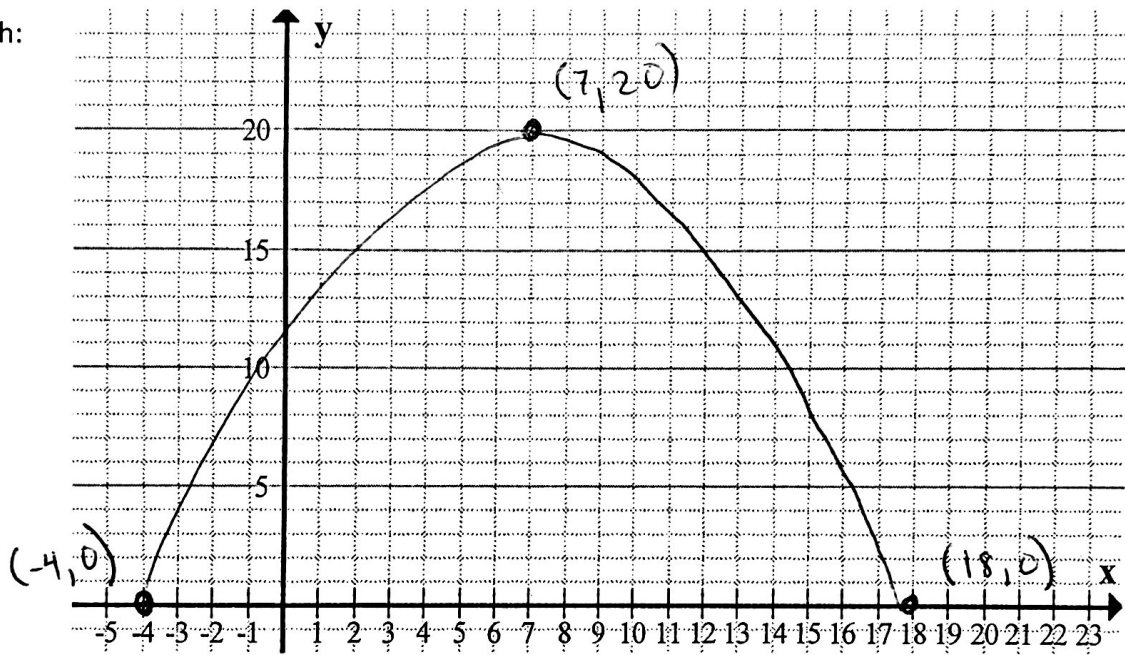
Factored Form of a Quadratic Relation | MFM2P

Example 2: The Dufferin Gate is a parabolic arch that is approximately 20 m tall and approximately 22 m wide.

a) Sketch a graph of the arch with the left base located on the x-axis 4 units to the left of the y-axis. Label the x-intercepts and vertex.

b) Determine an equation to model the arch.

Sketch:



Solution to a)

$$-4 + 22 = 18$$

$$\text{AOS: } \frac{-4 + 18}{2} = \frac{14}{2} = 7$$

Solution to b)

$$y = a(x-s)(x-t)$$

$$y = a(x+4)(x-18)$$

$$20 = a(7+4)(7-18)$$

$$20 = a(11)(-11)$$

$$20 = -121a$$

$$a = \frac{-20}{121}$$

$$\text{Equation: } y = \frac{-20}{121}(x+4)(x-18)$$

SUB IN
ZEROS.

SUB IN
(7,20)