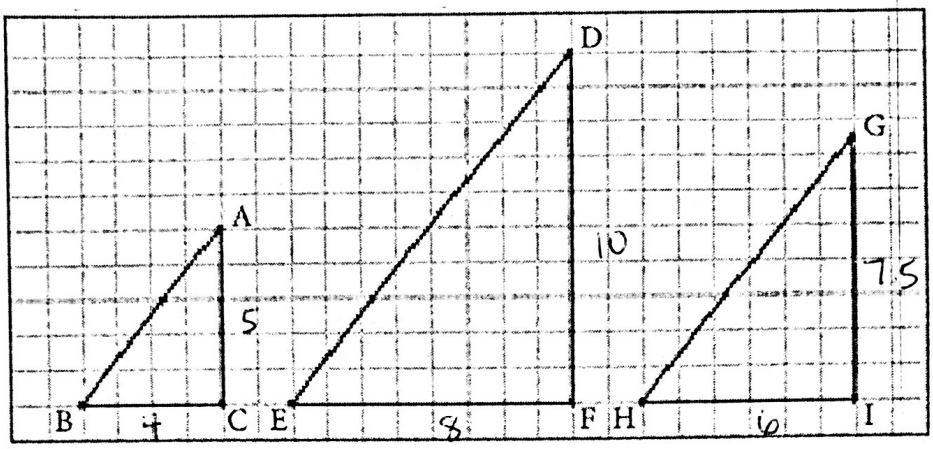
Consider the following three right angle triangles. They all have the same shape, so they are all similar triangles.



Find the lengths of the 3 hypotenuses using the Pythagorean Theorem. Keep you answer to 2 decimal places.

side AB:

$c^2 = a^2 + b^2$

$= 4^2 + 5^2$

$= 41$

$c = \sqrt{41}$

$c \doteq 6.4$

side DE:

$c^2 = 8^2 + 10^2$

$= 164$

$c = \sqrt{164}$

$c \doteq 12.81$

side GH:

$c^2 = 6^2 + 7.5^2$

$= 92.25$

$c = \sqrt{92.25}$

$c \doteq 9.6$

Now that we know all of the side lengths, we are going verify some properties of similar triangles.

We will first look at ΔABC and ΔDEF. Calculate the following 3 ratios:

$\frac{AB}{DE} = \frac{6.4}{12.81} \doteq 0.5$	$\frac{BC}{EF} = \frac{4}{8} = 0.5$	$\frac{AC}{DF} = \frac{5}{10} = 0.5$

Do the same thing for ΔDEF and ΔGHI. Calculate the following 3 ratios:

$\frac{DE}{GH} = \frac{12.81}{9.6} \doteq 1.33$	$\frac{EF}{HI} = \frac{8}{6} = 1.\overline{3}$	$\frac{DF}{GI} = \frac{10}{7.5} = 1.\overline{3}$

usually $k > 1$

KEY IDEA: Every pair of similar triangles has a "scale factor, k" that relates their corresponding sides:

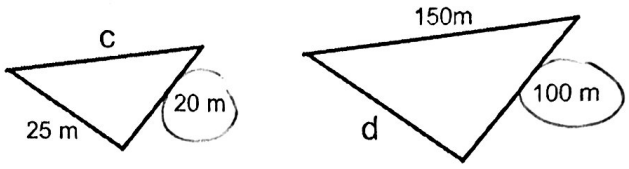
The scale factor between ΔABC and ΔDEF is: $k = 2$

The scale factor between ΔDEF and ΔGHI is: $k = 1.\overline{3}$ or $k = 4/3$

The scale factor between ΔGHI and ΔABC is: $k = 1.5$

We can use this fact, along with our solving proportions skills, to find unknown lengths in similar triangles. There are lots of practical applications of this.

Example: The following two triangles are similar...



What is the scale factor, k? $k = \frac{100}{20} = 5$

Use the scale factor to find these side measures...

side c: $150 \div 5 = 30m$

$c = 30m$

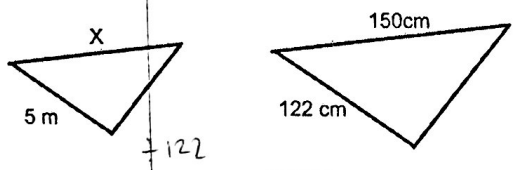
side d: $25 \times 5 = 125m$

$d = 125m$

Sometimes the answer can seem obvious, and you may be able to reason out an answer using mental math. These last examples here are not possible (or very difficult) with mental math or a scale factor, we will use a proportion.

Find the indicated side in each pair of similar triangles by solving a proportion:

a)



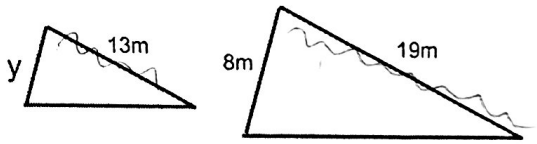
$\div 122$

$\frac{\text{Big}}{\text{Small}}$ $\frac{122}{5} = \frac{150}{x}$

$x = 5 \times 150 \div 122$

$x \doteq 6.15m$

b)



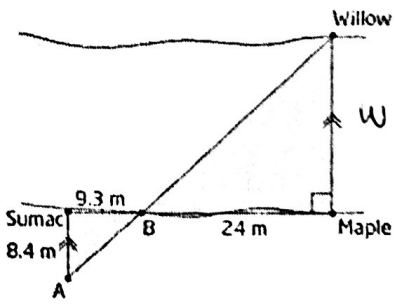
$\frac{\text{Big}}{\text{Small}}$ $\frac{19}{13} = \frac{8}{y}$

$y = 13 \times 8 \div 19$

$y = 5.47m.$

We can use similar triangles to solve problems, like those that arise in "surveying":

To determine the width of a river, Naomi finds a willow tree and a maple tree that are directly across from each other on opposite shores. Using a third tree on the shoreline, Naomi plants two stakes, A and B, and measures the distances shown. Find the width of the river using the information that Naomi found.



Let w be the width of the river

$\frac{\text{Big}}{\text{Small}}$ $\frac{24}{9.3} = \frac{w}{8.4}$

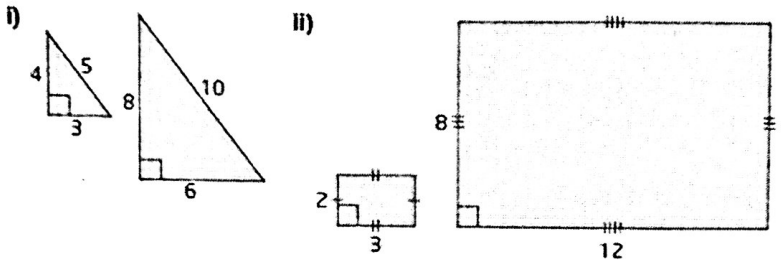
$w = 24 \times 8.4 \div 9.3$

$\boxed{w = 21.7m}$

The width is 21.7m.

Comparing Areas of Similar Figures:

Calculate the areas of the following figures. What is the relationship between their areas? What are the scale factors in each case?



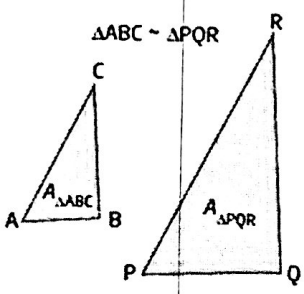
Small Triangle	Large Triangle	Small Rectangle	Large Rectangle
Area Calculation: $A = 3 \times 4 \div 2$ $A = 6$ units^2	Area Calculation: $A = 6 \times 8 \div 2$ $A = 24$ units^2	Area Calculation: $A = 12 \times 8$ $A = 96$ units^2	Area Calculation: $A = 2 \times 3$ $A = 6$ units^2
How are their areas related? $\frac{24}{6} = 4$ times as big		How are their areas related? $\frac{96}{6} = 16$ times as big	
What is the scale factor? $k = 2$		What is the scale factor? $k = 4$	

This relationship holds for ALL similar figures:

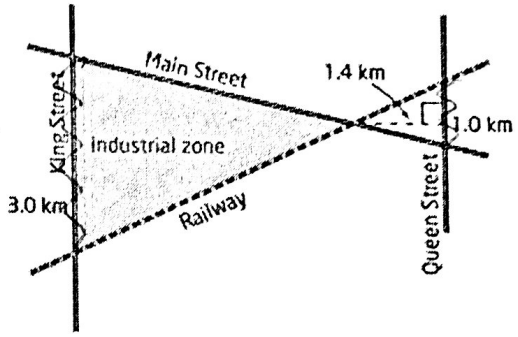
The ratio of areas in ~~simil~~ similar figures is the square of the scale factor, k^2.

Another way we can write this is:

$$A_{\Delta PQR} = k^2 A_{\Delta ABC}$$



Example: The shaded area is to be an industrial zone. Find the area of the industrial zone. Assume that King and Queen are parallel and that all streets and the track are straight.



Scale Factor: $k = \frac{3}{1} = 3$

$A_{small} = 1 \times 1.4 \div 2 = 0.7\ km^2$

$A_{large} = k^2 A_{small}$

$= 3^2(0.7)$

$= 9(0.7)$

$= 6.3\ km^2$

The industrial zone is $6.3\ km^2$.