Quadratic Relations Motivation | MPM2D

The school store decides to start selling cupcakes. Being in charge, you aren't sure exactly what to charge per cupcake, but you do know that you want to maximize the money you make.

Your teacher gave you this bit of market research:

"If you charge $2.00 per cupcake, you normally sell 10 every hour. Every time you lower the price by $0.10, you sell one more cupcake per hour. Every time you increase the price by $0.10, you sell one less cupcake per hour."

The question you need to ask is "What price should you charge per cupcake in order to make the most money possible?

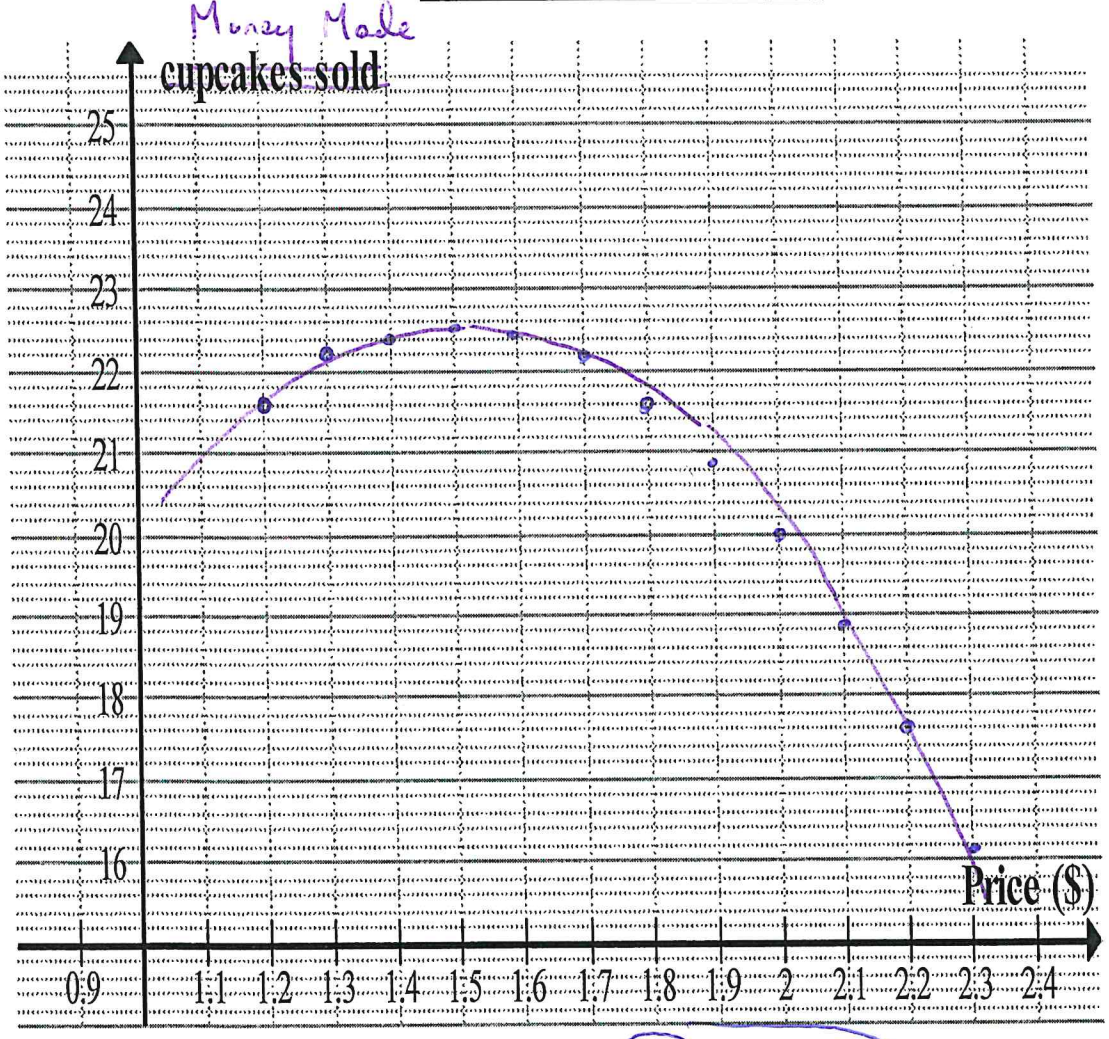
a) Fill in the table below to find out (start at 10 shirts and work up and down)

Price per Cupcake	Number of Cupcakes sold per hour	Total money made per hour (1^{st} × 2^{nd} columns)
$2.30	7	$16.10
$2.20	8	$17.60
$2.10	9	$18.90
$2.00	10	10 × $2.00 = $20.00
$1.90	11	$20.90
$1.80	12	$21.60
$1.70	13	$22.10
$1.60	14	$22.40
$1.50	15	$22.50
$1.40	16	$22.40
$1.30	17	$22.10
$1.20	18	$21.60

Conclusion: The maximum money of $22.50 is made per hour when you charge $1.50 per cupcake.

b) Plot the data from the other side on the grid below:

Cupcake Price vs. Money Made



c) Is this relationship a linear relationship or a non-linear relationship?

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Investigation: Quadratic Relations

For the following 4 relations, describe the written rule, generate points using your table of values, and draw each line. When you've drawn the line, read off the slope of the line, and the y-intercept.

Relation #1

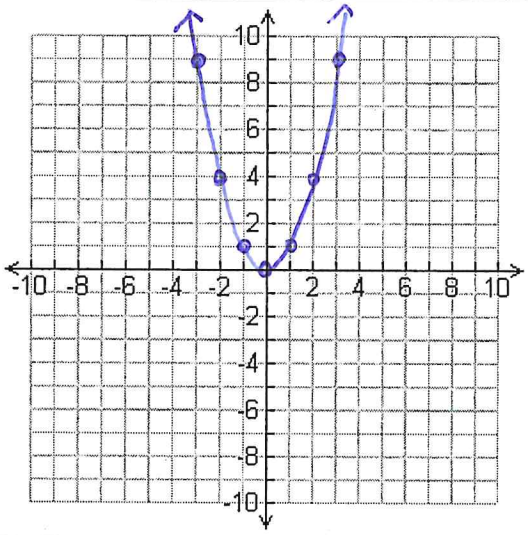
Equation: $y = x^2$

Written Rule:

The y-values are the square of the x-values.

x	y
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Graph:



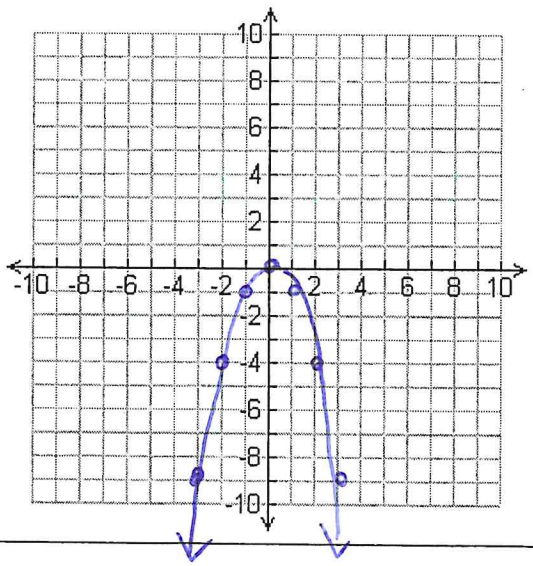
Relation #2

Equation: $y = -x^2$

Written Rule: The y-values are the negatives of the square of the x-values.

x	y
-3	$-(-3)^2 = -9$
-2	-4
-1	-1
0	0
1	-1
2	-4
3	-9

Graph:



Relation #3

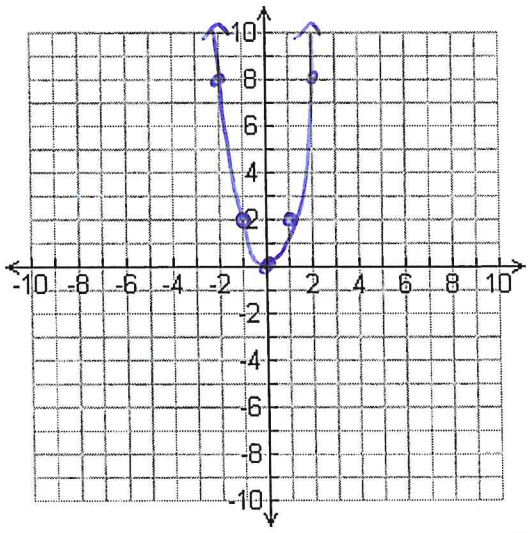
Equation: $y = 2x^2$

Written Rule:

The y-values are double the square of the x-values

x	y
-3	18
-2	8
-1	2
0	0
1	2
2	8
3	18

Graph:



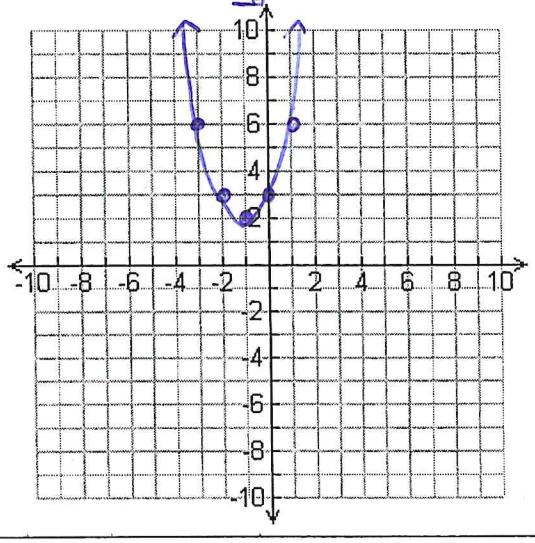
Relation #4

Equation: $y = x^2 + 2x + 3$

Written Rule: The y-values are the squares of the x-values, plus double the x-values, plus 3.

x	y
-3	$9 - 6 + 3 = 6$
-2	$4 - 4 + 3 = 3$
-1	$1 - 2 + 3 = 2$
0	$0 + 0 + 3 = 3$
1	$1 + 2 + 3 = 6$
2	$4 + 4 + 3 = 11$
3	$9 + 6 + 3 = 18$

Graph:



Each of these graphs represents a quadratic relation. Over the next few days we will be exploring:

- Properties of Quadratic Relations
- Different Forms of Quadratic Relations
- How to Graph Quadratic Relations

We will then further explore exponents, and the power laws you learned about it grade 9.

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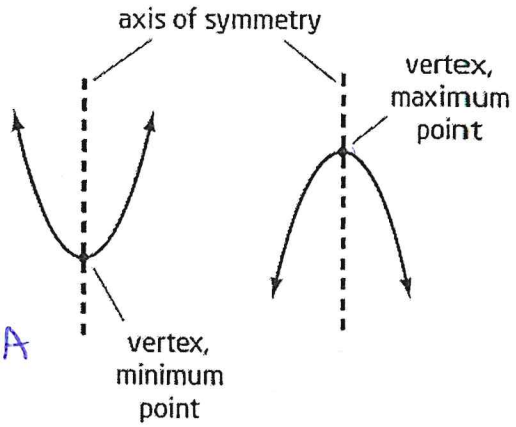
Summary:

1) Any equation of the form $y = ax^2 + bx + c$ is a quadratic relation.

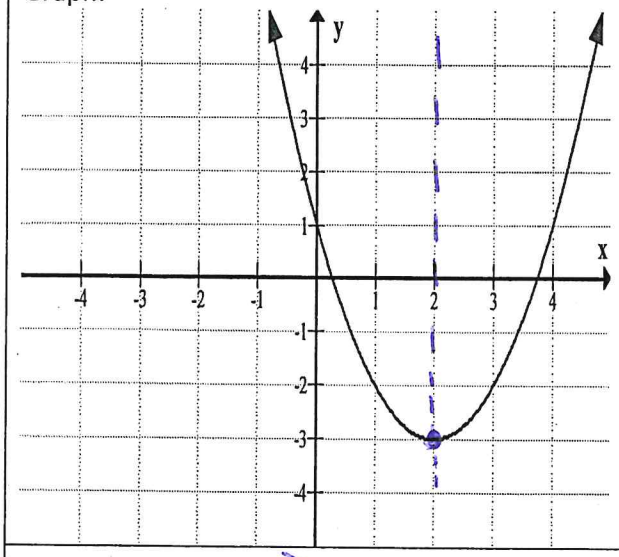
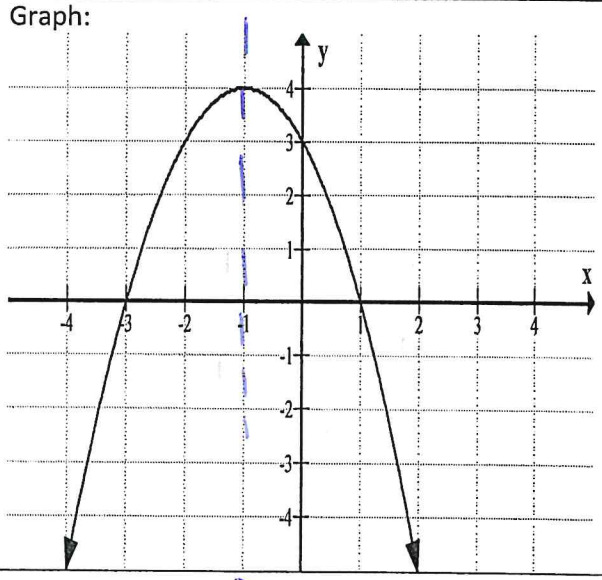
2) The graph of a quadratic relation is called a PARABOLA



3) The vertex is the highest or lowest point on the parabola.

4) The vertical line that divides the parabola in half is called the axis of symmetry. It always passes through the vertex.

Example: What is the vertex and axis of symmetry for the following two quadratic relations?

Graph:	Graph:
	
Vertex: (2, -3)	Vertex: (-1, 4)
Axis of Symmetry: $x = 2$	Axis of Symmetry: $x = -1$

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How can you use finite differences to tell if a relation is linear or quadratic? Mr. Smith will do one example with you , and then you will calculate the finite differences for the remaining relations.

a) $y = 2x - 5$

x	y	First Differences
-2	-9	
-1	-7	-7 - (-9) = 2
0	-5	-5 - (-7) = 2
1	-3	2
2	-1	2

b) $y = -6x + 2$

x	y	First Differences
-2	14	
-1	8	8 - 14 = -6
0	2	2 - 8 = -6
1	-4	-6
2	-10	-6

c) $y = x^2 - 4$

x	y	First Differences	Second Differences
-2	0		
-1	-3	-3 - 0 = -3	
0	-4	-4 - (-3) = -1	-1 - (-3) = 2
1	-3	-3 - (-4) = 1	1 - (-1) = 2
2	0	0 - (-3) = 3	3 - 1 = 2

d) $y = 2x^2 + 3x - 1$

x	y	First Differences	Second Differences
-2	1		
-1	-2	-3	4
0	-1	1	4
1	4	5	4
2	13	9	

$2(2)^2 + 3(2) - 1$
$= 8 + 6 - 1$
$= 13$

Can you tell what is true about the finite differences of a linear relation?

The first differences are equal.

Can you tell what is true about the finite differences of a quadratic relation?

The second differences are equal.