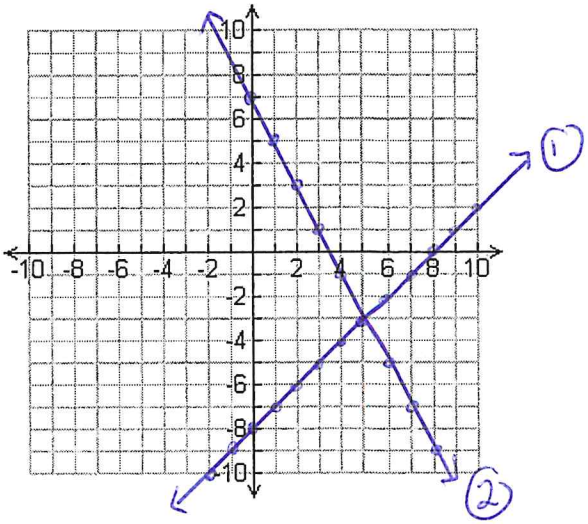
The Method of Substitution: Part 2 | MPM2D

Today we will be learning a purely **algebraic** method for solving linear systems when our lines are not in $y = mx + b$ form.



1) Consider the following linear system. Rearrange the second equation, and solve this linear system **by graphing**.

$y = x - 8$ ①

$2x + y = 7$ ② $y = -2x + 7$

Point of Intersection = $(5, -3)$

2) Consider again the same linear system. Using your rearranged version of equation ②, use yesterday's method to find the point of intersection.

Set ① = ②

$x - 8 = -2x + 7$
$+2x \quad +2x$
$3x - 8 = 7$
$+8 \quad +8$
$3x = 15$
$\boxed{x = 5}$

Sub in ①

$y = 5 - 8$
$\boxed{y = -3}$

In practice, both equations may not be in $y = mx + b$ form and it may be very impractical to use this method. We will now solve this linear system a third way by using a more general substitution method.

KEY IDEA: Isolate a variable in one of the equations and substitute it into the other.

Use this key idea (substituting equation ① into equation ②) to solve the above linear system.

Sub ① in ②

$2x + x - 8 = 7$
$3x - 8 = 7$
$+8 \quad +8$
$3x = 15$
$\boxed{x = 5}$

sub $x = 5$ in ①

$y = 5 - 8$
$\boxed{y = -3}$

Example: Use our new method to solve the following linear system.

$y = -3x + 4$ ①

$2x - 3y = 32$ ②

sub ① in ②

$2x - 3(-3x + 4) = 32$

$2x + 9x - 12 = 32$

$11x - 12 = 32$

$+12 \quad +12$

$11x = 44$

$\boxed{x = 4}$

sub $x = 4$ in ①

$y = -3(4) + 4$

$\boxed{y = -8}$

POI is $(4, -8)$

If both equations are not in $y = mx + b$ form you will have to choose a variable to isolate. In this next example, let's isolate for x in the second equation.

$2x + 3y = 9$ ①

$x - 2y = 1$ ② $\rightarrow x = 2y + 1$

sub $y = 1$ in ②

$x = 2(1) + 1$

$\boxed{x = 3}$

Sub ② in ①

$2(2y + 1) + 3y = 9$

$4y + 2 + 3y = 9$

$7y + 2 = 9$

$-2 \quad -2$

$7y = 7$

$\boxed{y = 1}$

You try it: Decide which variable you are going to isolate in the following linear system, and try our new method.

$2x + 5y = 7$ ①

$4x + y = 5$ ② $\rightarrow y = -4x + 5$

Sub ② in ①

$\cancel{2(-4x+5) + 5y}$

$2x + 5(-4x + 5) = 7$

$2x - 20x + 25 = 7$

$-18x + 25 = 7$

$-25 \quad -25$

$-18x = -18$

$\boxed{x = 1}$

Sub $x = 1$ in ②

$y = -4(1) + 5$

$\boxed{y = 1}$

Practice

Solve the following linear systems using our new method.

1) $y = 3x - 5$ ①

$2x + 5y = 9$ ②

Sub ① in ②

$2x + 5(3x - 5) = 9$

$2x + 15x - 25 = 9$

$17x - 25 = 9$

$17x = 34$

$\boxed{x = 2}$

Sub in ①

$y = 3(2) - 5$

$\boxed{y = 1}$

2) $x = 4y - 7$ ①

$5x + 2y = 31$ ②

Sub ① in ②

$5(4y - 7) + 2y = 31$

$20y - 35 + 2y = 31$

$22y - 35 = 31$

$22y = 66$

$\boxed{y = 3}$

Sub in ①

$x = 4(3) - 7$

$\boxed{x = 5}$

3) $x = 6y - 1$ ①

$4x - 2y = -26$ ②

Sub ① in ②

$4(6y - 1) - 2y = -26$

$24y - 4 - 2y = -26$

$22y - 4 = -26$

$22y = -22$

$\boxed{y = -1}$

Sub in ①

$x = 6(-1) - 1$

$\boxed{x = -7}$

4) $x - 3y = 2$ ① $\rightarrow x = 3y + 2$

$5x + y = 26$ ② $\rightarrow y = -5x + 26$

a) Solve this system by isolating x in equation ①

$5(3y + 2) + y = 26$

$15y + 10 + y = 26$

$16y + 10 = 26$

$16y = 16$

$\boxed{y = 1}$

Sub in ① $\rightarrow x = 3(1) + 2$

$\boxed{x = 5}$

b) Solve this system by isolating y in equation ②

$x - 3(-5x + 26) = 2$

$x + 15x - 78 = 2$

$16x - 78 = 2$

$16x = 80$

$\boxed{x = 5}$

Sub in ② $\rightarrow y = -5(5) + 26$

$\boxed{y = 1}$

5) $2x + 3y = 10$ ①

$x - 5y = -8$ ② $\rightarrow x = 5y - 8$

Sub in ① $2(5y - 8) + 3y = 10$

$10y - 16 + 3y = 10$

$13y - 16 = 10$

$13y = 26$

$\boxed{y = 2}$

Sub in ② $x = 5(2) - 8$

$\boxed{x = 2}$

6) $5x - y = 10$ ① $\rightarrow -y = -5x + 10$

$4x + 3y = 46$ ② $y = 5x - 10$

sub in ② $4x + 3(5x - 10) = 46$

$4x + 15x - 30 = 46$

$19x - 30 = 46$

$19x = 76$

$\boxed{x = 4}$

sub in ① $y = 5(4) - 10$

$\boxed{y = 10}$

Solutions: 1) $(2, 1)$ 2) $(5, 3)$ 3) $(-7, -1)$ 4) $(5, 1)$ 5) $(2, 2)$ 6) $(4, 10)$