The Sine and Cosine Ratios | MPM2D

Today we are going to look at two other ratios within similar right angle triangles. Specifically, the ratios involving the hypotenuse. Mr. Smith will put some similar triangles up on the screen, and you will calculate some ratios.

Angle of 50°					
Rise (opposite)	Run (adjacent)	Hypotenuse	Tangent Ratio	$^{opp}/_{hyp}$	$^{adj}/_{hyp}$
7.76	6.52	10.14	1.1902	0.7653	0.6430
13.19	11.08	17.22	1.1904	0.7660	0.6434
3.69	3.1	4.82	1.1903	0.7656	0.6432

Given a certain angle, not just the Tangent ratio, but the other two ratios are the same too, no matter how big or small the triangle. We give the final two ratios special names as well. These form the three primary trigonpmetric ratios:

The Tangent Ratio = $\frac{opp}{adj}$ The Sine Ratio = $\frac{opp}{hyp}$ The Cosine Ratio = $\frac{adj}{hyp}$

Based on our above calculations, we would say...

The tangent ratio of 50° is approximately = 1.19

The sine ratio of 50° is approximately = 0.77

The cosine ratio of 50° is approximately = 0.64

primary trigonometric ratios
- sine, cosine, and tangent
- often abbreviated as sin, cos, and tan

We said approximately here, because our measurements were not quite precise. We can find precise values of these tangent ratios by evaluating the following (to 4 decimal places):

$tan50° =$ 1.1918 $sin50° =$ 0.7660 $cos50° =$ 0.6428

Find the following ratios for other angles using your calculator (to 4 decimal places):

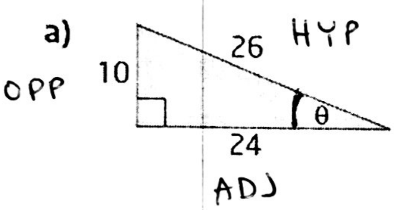
$sin45° =$ 0.7071 $cos61° =$ 0.4848 $sin85° =$ 0.9962

$cos5° =$ 0.9962 $cos88° =$ 0.0349 $cos89° =$ 0.0175

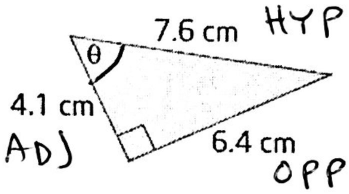
Example:

Find the three primary trigonometric ratios for θ. Express the ratios as decimals, rounded to four decimal places.

a)



b)



Part a)		
$sin\theta = \frac{opposite}{hypotenuse}$ $= 10/26$ $= 0.3846$	$cos\theta = \frac{adjacent}{hypotenuse}$ $= 24/26$ $= 0.9231$	$tan\theta = \frac{opposite}{adjacent}$ $= 10/24$ $= 0.4167$
Part b)		
$sin\theta = \frac{opposite}{hypotenuse}$ $= 6.4/7.6$ $= 0.8421$	$cos\theta = \frac{adjacent}{hypotenuse}$ $= 4.1/7.6$ $= 0.5395$	$tan\theta = \frac{opposite}{adjacent}$ $= \frac{6.4}{4.1}$ $= 1.5610$

A memory device (or mnemonic) for the three primary trigonometric ratios uses these short forms:

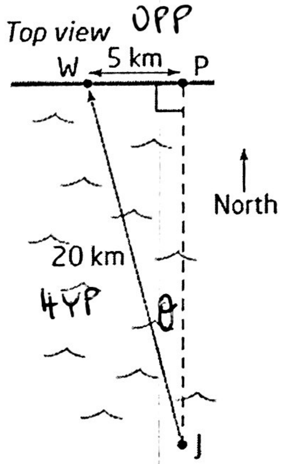
$S = \frac{O}{H}$ $C = \frac{A}{H}$ $T = \frac{O}{A}$

These short forms produce the nonsense phrase soh cah toa. This phrase may help you remember the formulas for the trigonometric ratios.

Example: Finding angles with the sine and cosine ratios.

In practice, you will decide on your own (based on what information you have) whether sine, cosine, or tangent will help you solve a problem.

a) Captain Jack is navigating his ship to Port Harbour, which is directly north of the ship's location. To compensate for an easterly current, he aims for a point on shore that is 5 km west of Port Harbour. Assuming that the point on shore is 20 km from his position now, at what bearing must Jack head his ship?



$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = 5/20$

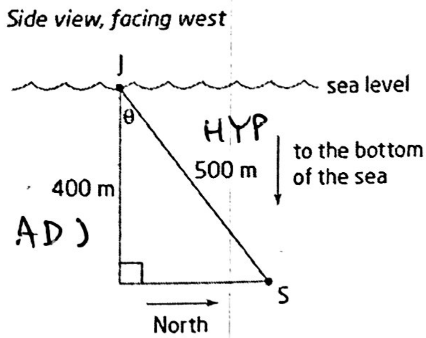
$\sin\theta = 0.25$

$\theta = \sin^{-1}(0.25)$

$\boxed{\theta = 14.5^\circ}$

He must head 14.5° west of north.

b) Captain Jack is in communication with a submarine that is cruising at a depth of 400 m below sea level. If Jack's radar tells him that the submarine is 500 m from Jack, due north of his ship, at what angle is the submarine located with respect to Captain Jack's ship, to the nearest degree?



$\cos\theta = \frac{adj}{hyp}$

$\cos\theta = 400/500$

$\cos\theta = 0.8$

$\theta = \cos^{-1}(0.8)$

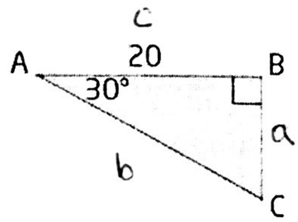
$\boxed{\theta = 36.9^\circ}$

The submarine is 36.9° from vertical.

Example: Solving a triangle

To solve a triangle means to find out every single thing about that triangle, all the sides and all the angles.

Solve △ABC. Round side lengths to the nearest unit and angles to the nearest degree.



Let's choose a strategy first:

1) Find C (angles add to 180°)
2) Find side a using "tan"
3) Find side b using Pyth. Thm.

Literacy Connections

When naming parts of triangles, use capital letters to represent the angles and vertices and corresponding lowercase letters to represent opposite sides.

1) $C = 180 - 90 - 30$

$\boxed{C = 60^\circ}$

2) $\frac{\tan 30^\circ}{1} = \frac{a}{20}$

$a = 20 \tan 30^\circ$

$\boxed{a \doteq 11.5}$

3) $b^2 = 20^2 + 11.5^2$

$b^2 = 532.25$

$\boxed{b \doteq 23.1}$