Recall that trigonometry means... triangle measurement

In this unit, we used a variety of tools to find sides and angles in all types of triangles.

Two useful tools that we will not cover in this note are:

- Pythagorean Theorem
- Angle Sum of a Triangle Theorem (180°)

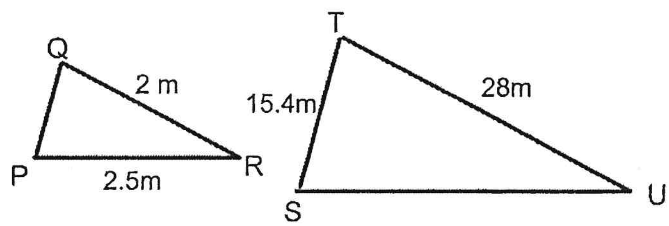
You should have these in your math toolbox, ready to use at any time.

We began by talking about similar triangles.

**Similar Triangles -** Triangles with the same shape (angles). Ratios of corresponding sides are equal.

**Scale Factor -** (k) how much bigger the large sides were compared to the small sides.

We used proportional reasoning to find unknown sides in similar triangles. For example, the two triangles in this diagram are similar!




a) Find side SU using the scale factor

$k = \frac{28}{2} = 14$

$SU = 14 \times 2.5$

$\boxed{SU = 35m}$

b) Find side PQ using a proportion

$\frac{Big}{Small} = \frac{28}{2} = \frac{15.4}{PQ}$

$PQ = 2 \times 15.4 \div 28$

$\boxed{PQ = 1.1m}$

c) The area of triangle PQR is 1.25 m$^2$. What is the height of triangle STU?

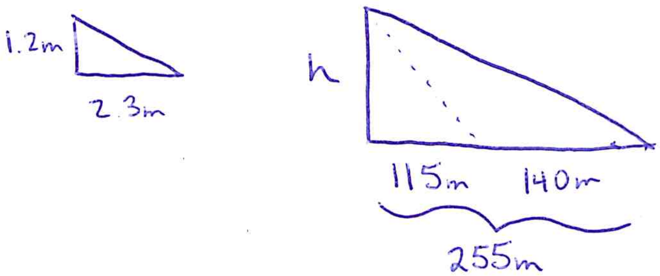
The areas are related by $k^2 = 14^2 = 196$

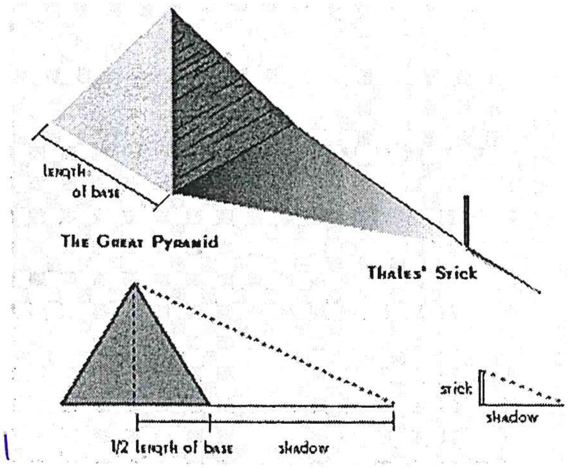
$A_{STU} = 1.25 \times 196$

$= 245m^2$

We also used similar triangles to problem solve. Similar triangles can be used to measure inaccessible heights.

Example: The mathematician and philosopher Thales measured the heights of the pyramids using similar triangles formed by shadows. The Great Pyramid had a base length of 230m. It's shadow was 140m long at a certain time of day. At the same time, Thales had a 1.2m high stick that cast a shadow 2.3m long. How tall is the great pyramid?




$$\frac{Big}{Small} = \frac{255}{2.3} = \frac{h}{1.2}$$

$h = 255 \times 1.2 \div 2.3$

$h = 133.04$m tall

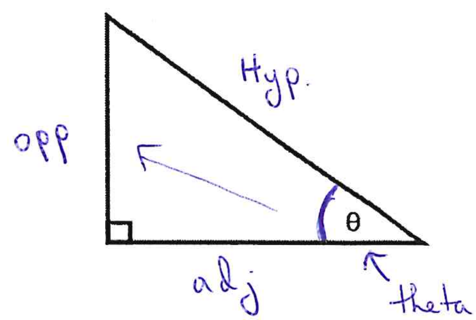
We then turned our attention to ratios within triangles. We looked at three special ratios:

**The Three Primary Trig Ratios (SOHCAHTOA)**

Recall for any right-angle triangle:

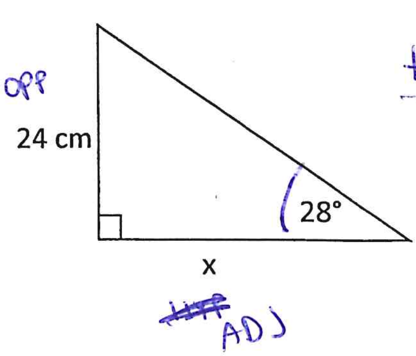
$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$

$\tan\theta = \frac{opp}{adj}$ $S\frac{O}{H}C\frac{A}{H}T\frac{O}{A}$




Examples: Determine the measurement of the indicated variable.

a)



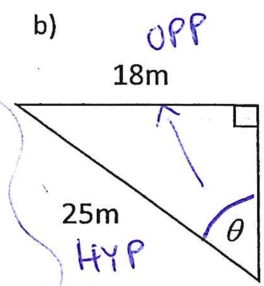

$\tan\theta = \frac{opp}{adj}$

$\frac{\tan 28°}{1} = \frac{24}{x}$

$x = 1 \times 24 \div \tan 28$

$\boxed{x = 45.1\text{cm}}$

b)




$\sin\theta = \frac{opp}{hyp}$

$\sin\theta = \frac{18}{25}$

$\sin\theta = 0.72$

$\theta = \sin^{-1}(0.72)$

$\boxed{\theta = 46.1°}$

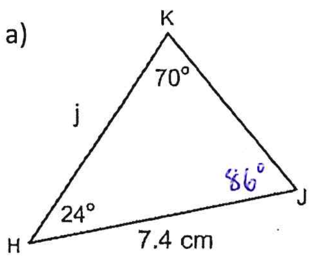
Finally, we developed tools to solve acute triangles (triangles with all angles less than 90°)

***The Sine Law:*** $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ OR $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

When do I use it? When you have a side-angle pair

Examples: Determine the measure of the indicated variable.

a)



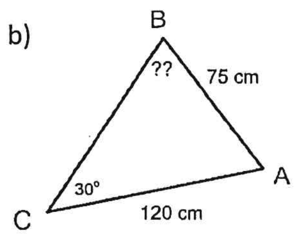

$\frac{j}{\sin J} = \frac{k}{\sin K}$

$\frac{j}{\sin 86°} = \frac{7.4}{\sin 70°}$

$j = \frac{7.4 \sin 86°}{\sin 70°}$

$\boxed{j = 7.86 cm}$

b)




$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin B}{120} = \frac{\sin 30}{75}$

$\sin B = \frac{120 \sin 30}{75}$

$\sin B = 0.8$

$\boxed{B = 53.1°}$

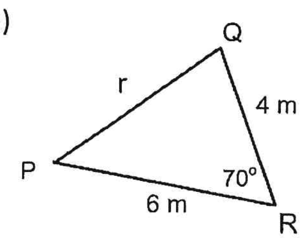
***The Cosine Law:***

$c^2 = a^2 + b^2 - 2ab \cdot \cos C$ OR $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

When do I use it?

Examples: Determine the measurement of the indicated variable.

a)




$r^2 = p^2 + q^2 - 2pq \cdot \cos R$

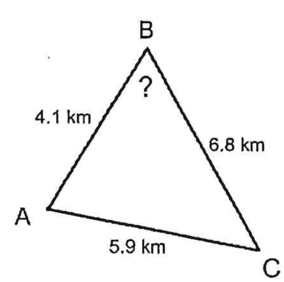
$= 4^2 + 6^2 - 2(4)(6)\cos 70$

$= 16 + 36 - 16.4$

$= 35.6$

$\boxed{r \doteq 5.97 m}$

b)




$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cancel{= 5.9^2 + 4.1^2}$

$= \frac{6.8^2 + 4.1^2 - 5.9^2}{2(6.8)(4.1)}$

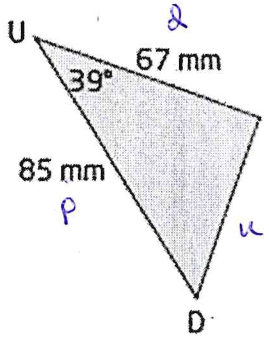
$\cos B = \frac{28.24}{55.76}$

$\cos B = 0.5065$

$\boxed{B = 59.6°}$

We then spent time problem solving, and solving triangles. Here, you had to decide on your own what trigonometry tool would help you.

Example: Solve the following triangle




① $u^2 = d^2 + p^2 - 2dp \cdot \cos U$

$= 67^2 + 85^2 - 2(67)(85)\cos 39$

$= 2,862.3$

$\boxed{u \doteq 53.5\text{ mm}}$

③ $D = 180 - 39 - 52$

$\boxed{D = 89°}$

② $\frac{\sin D}{d} = \frac{\sin U}{u}$

$\frac{\sin D}{67} = \frac{\sin 39}{53.5}$

$\sin D = \frac{67 \sin 39}{53.5}$

$\sin D = 0.7881$

$\boxed{D = 52°}$

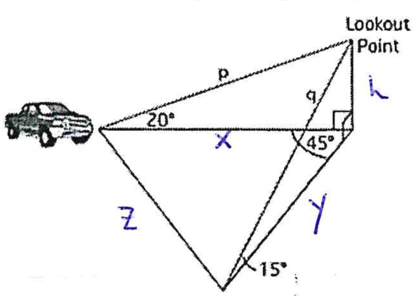
Example: Lookout Point is accessible from two trails, both of which start from the same altitude and climb upward. Path $p$ travels east to the point and climbs at an average angle of elevation of 20°. Path $q$ travels northeast to the point at an average angle of elevation of 15°. Path $p$ is 2.0 km long. Jack and Debbie parked at the base of path $p$. They hiked a round trip up path $p$ to Lookout Point, then down path $q$, and then finally straight from the base of path $q$ back to their truck. How far did they hike, to the nearest tenth of a kilometre? State any assumptions you make.

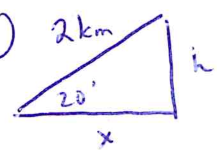
① Find x & h

② Find y

③ Find z

④ Add p, q, z

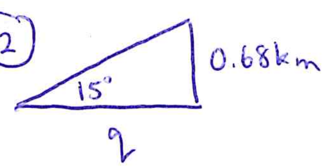



① 


$\sin 20° = \frac{h}{2}$ $\cos 20 = \frac{x}{2}$

$h = 2\sin 20$ $x = 2\cos 20$

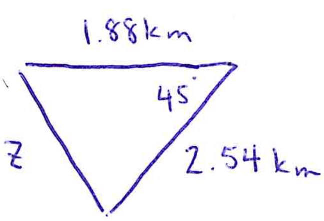
$\boxed{h = 0.68\text{ km}}$ $\boxed{x = 1.88\text{ km}}$

② 


$\frac{\tan 15°}{1} = \frac{0.68}{q}$

$q = 1 \times 0.68 \div \tan 15$

$\boxed{q = 2.54\text{ km}}$

③ 


$z^2 = 1.88^2 + 2.54^2 - 2(1.88)(2.54)\cos 45$

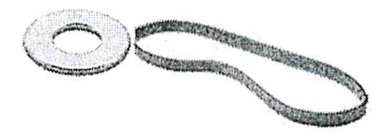
$z^2 = 3.23$

$\boxed{z = 1.8\text{ km}}$

Extra Text Review Questions: page 434 #1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

Total hiked = $p + q + z = 2 + 2.54 + 1.8 = \boxed{4.34\text{ km}}$

# Performance Task Prep: Washer Launchers | MPM2D



In this task, you will be making a simple launcher out of a rubber band and a washer. You will be calibrating your rubber band, measuring how fast the washer launches given a certain amount of stretch. You will then be making predictions on the maximum height, flight time, and range of your washer given different launch angles.

Preparation Questions:

1) A student launches their washer launcher from a meter stick, straight up into the air. The height of the washer (in meters) over time (seconds) is given by:

$$h = -4.9t^2 + bt + 1$$

Where "b" is the unknown initial vertical speed of the washer. The student measures a flight time of 1.26 seconds before the washer hits the ground. Calculate the initial speed of the washer.

$h = 0$ when $t = 1.26$

$0 = -4.9(1.26)^2 + 1.26b + 1$

$0 = -7.77924 + 1.26b + 1$

$0 = -6.77924 + 1.26b$

$6.77924 = 1.26b$

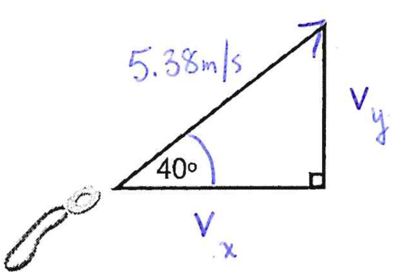
$b = 5.38$ m/s

Initial vertical speed = 5.38 m/s

2) The student wants to launch their washer at an angle of 40° and predict several things about the flight path:

- The flight time
- The maximum height
- The distance traveled

a) If the launcher is launched at a 40° angle, with the same launch speed in question 1), calculate the initial vertical speed of the launcher, and the horizontal speed of the launcher.




i) vertical speed

$\sin 40° = \dfrac{V_y}{5.38}$

$V_y = 5.38 \sin 40°$

$V_y = 3.46$ m/s.

ii) horizontal speed

$\cos 40° = \dfrac{V_x}{5.38}$

$V_x = 5.38 \cos 40°$

$V_x = 4.12$ m/s.

# Performance Task Prep: Washer Launchers | MPM2D

b) Determine an equation for the height of the washer over time if it is launched at this angle, from a height of 1m.

$$h = -4.9t^2 + 3.46t + 1$$

c) Using your equation, determine the flight time and maximum height of the washer (to 2 decimal places).

Flight time calculation: QF

$a = -4.9 \quad b = 3.46 \quad c = 1$

$$t = \frac{-3.46 \pm \sqrt{3.46^2 - 4(-4.9)(1)}}{2(-4.9)}$$

$$= \frac{-3.46 \pm \sqrt{31.57}}{-9.8}$$

$t_1 = -0.22\text{s}$ INADMISSABLE

$t_2 = 0.93\text{s}$ ↑ Lands in 0.93s.

Maximum height calculation:

$$t = \frac{-b}{2a}$$

$$t = \frac{-3.46}{2(-4.9)}$$

$t = 0.35\text{ s}$

$$h = -4.9(0.35)^2 + 3.46(0.35) + 1$$

$= 1.61\text{ m}$

d) Using the horizontal speed from a) and the flight time from c), determine how far the washer flew, and summarize your findings on the diagram given.

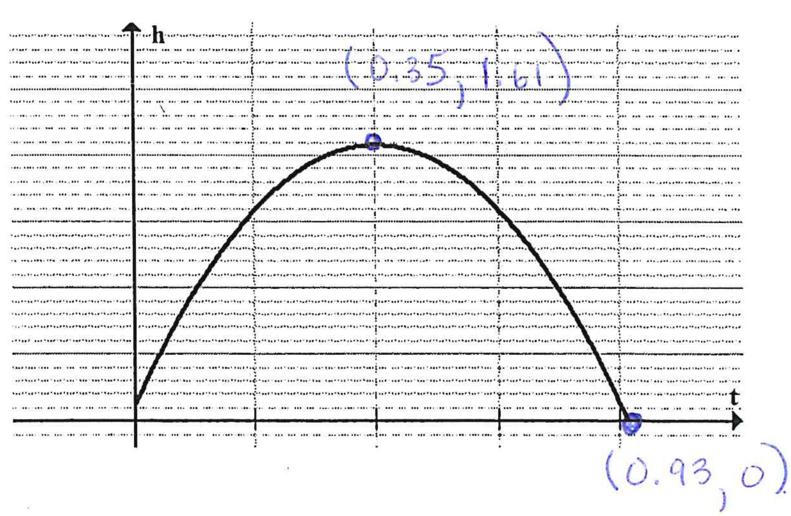
Hor. Speed = 4.12 m/s

Flight time = 0.93s

dist. = speed × time

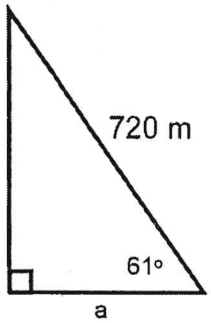
= 4.12 × 0.93

= 3.83 m



1) Determine the length of the indicated measure to one decimal place.

a)

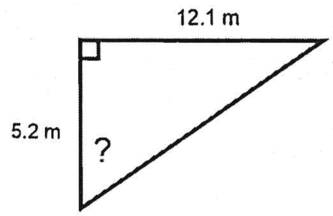



$\cos 61^\circ = \frac{a}{720}$

$a = 720 \cos 61^\circ$

$\boxed{a = 349.1\text{m}}$

b)



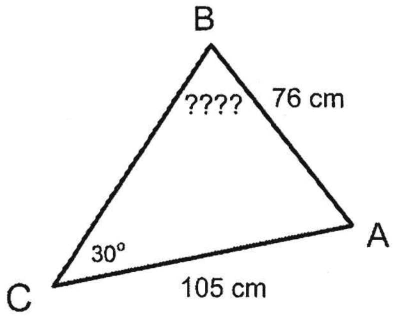

$\tan\theta = \frac{12.1}{5.2}$

$\tan\theta = 2.3269$

$\theta = \tan^{-1}(2.3269)$

$\boxed{\theta = 66.7^\circ}$

c)



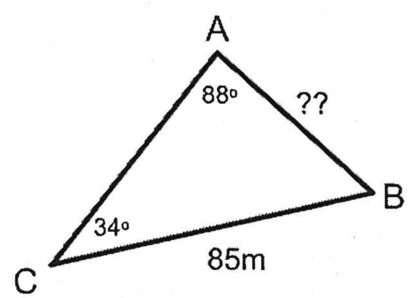

$\frac{\sin B}{105} = \frac{\sin 30}{76}$

$\sin B = \frac{105 \sin 30}{76}$

$\sin B = 0.6908$

$\boxed{B = 43.7^\circ}$

d)

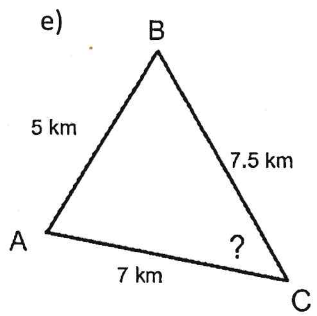



$\frac{c}{\sin 34} = \frac{85}{\sin 88}$

$c = \frac{85 \sin 34}{\sin 88}$

$\boxed{c = 47.6\text{m}}$

e)



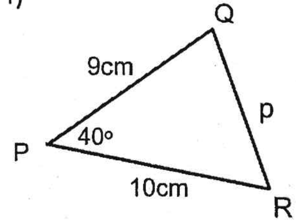

$\cos C = \frac{7^2 + 7.5^2 - 5^2}{2(7)(7.5)}$

$\cos C = \frac{80.25}{105}$

$\cos C = 0.7643$

$\boxed{C = 40.2^\circ}$

f)




$p^2 = 9^2 + 10^2 - 2(9)(10)\cos 40$

$p^2 = 81 + 100 - 137.9$

$p^2 = 43.1$

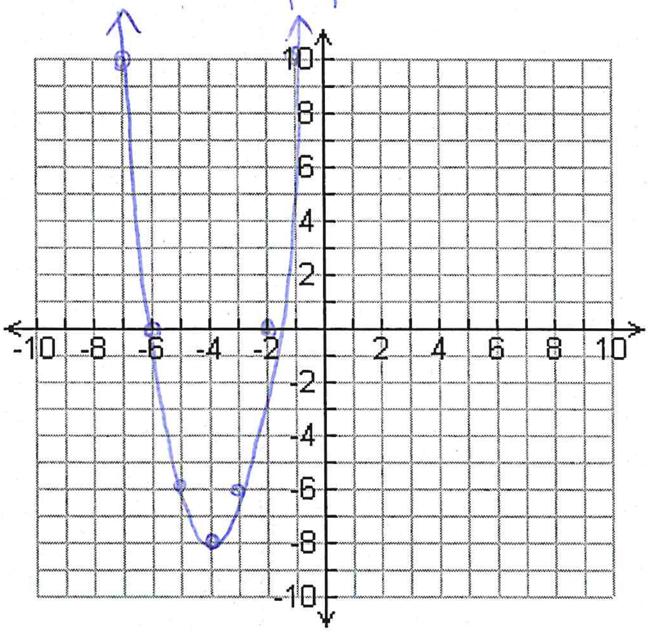
$\boxed{p = 6.6\text{cm}}$

2) Make a sketch of the following quadratic relations.

a) $y = 2(x+4)^2 - 8$

Vertex: $(-4, -8)$

Step Pattern: 2, 6, 10



b) $y = 2(x-1)(x+3)$

Zeros: $(1,0)$ & $(-3,0)$

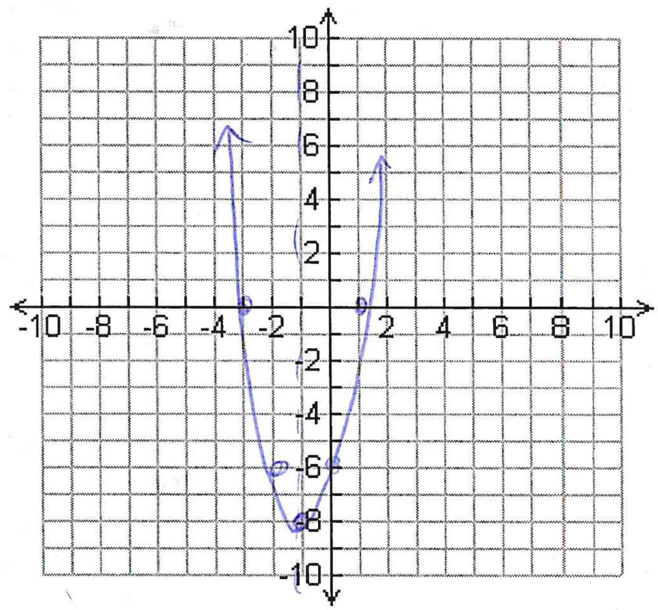
Step Pattern: 2, 6, 10

AOS: $x = -1$

$y = 2(-1-1)(-1+3)$

$= 2(-2)(2)$

$= -8$



3) Factor the following fully:

a) $x^2 - 7x - 18$

(×) −18
(+) −7

$= (x-9)(x+2)$

b) $4x^2 - 121$

DOS

$= (2x-11)(2x+11)$

c) $2x^2 + 6x + 4$

$= 2(x^2+3x+2)$

(×) 2
(+) 3

$= 2(x+2)(x+1)$

d) $3x^2 - 8x + 4$

(×) 12
(+) −8

$= 3x^2 - 2x - 6x + 4$

$= x(3x-2) - 2(3x-2)$

$= (3x-2)(x-2)$

e) $9x^2 - 30x + 25$

$= (3x-5)^2$

$\sqrt{9x^2} = 3x$

$\sqrt{25} = 5$

$2(3x)(5) = 30x$