Zero and Negative Exponents | MPM2D

Today we are going to investigate what happens when an exponent is zero, or negative. In this investigation, we are going to evaluate expressions with powers two ways to (hopefully) see a rule for these special exponents.

In the middle column, expand each power and simplify. In the last column, use the exponent laws to simplify the expression into a single power. **Leave your answers as fractions!**

Expression to be simplified	Expanded Form	Using Exponent Laws	Conclusion
$\frac{2^3}{2^1}$	$\frac{2\times2\times2}{2}=\frac{8}{2}=4$	$2^{3-1}=2^2$	$2^2=4$
$\frac{2^3}{2^2}$	$\frac{2\times2\times2}{2\times2}=\frac{8}{4}=2$	$2^{3-2}=2^1$	$2^1=2$
$\frac{2^3}{2^3}$	$\frac{2\times2\times2}{2\times2\times2}=\frac{8}{8}=1$	$2^{3-3}=2^0$	$2^0=1$
$\frac{2^3}{2^4}$	$\frac{2\times2\times2}{2\times2\times2\times2}=\frac{8}{16}=\frac{1}{2}$	$2^{3-4}=2^{-1}$	$2^{-1}=\frac{1}{2}$
$\frac{2^3}{2^5}$	$\frac{2\times2\times2}{2\times2\times2\times2\times2}=\frac{8}{32}=\frac{1}{4}$	$2^{3-5}=2^{-2}$	$2^{-2}=\frac{1}{4}$
$\frac{2^3}{2^6}$	$\frac{2\times2\times2}{2\times2\times2\times2\times2\times2}=\frac{8}{64}=\frac{1}{8}$	$2^{3-6}=2^{-3}$	$2^{-3}=\frac{1}{8}$

Rule for Zero Exponents: $a^0 = 1$ except for $a=0$

Why does it work?

Anything divided by itself is 1

Ex/ $\frac{22^7}{22^7} = 22^0 = 1$

Rule for Negative Exponents: $a^{-b} = 1/a^b$

Why does it work? Negative exponents can be thought of as dividing a smaller power by a larger power.

Visual Summary (Powers of 2):

2^{-4}	2^{-3}	2^{-2}	2^{-1}	2^{0}	2^{1}	2^{2}	2^{3}	2^{4}
$1/16$	$1/8$	$1/4$	$1/2$	1	2	4	8	16

You complete this visual summary for powers of 3:

3^{-4}	3^{-3}	3^{-2}	3^{-1}	3^{0}	3^{1}	3^{2}	3^{3}	3^{4}
$1/81$	$1/27$	$1/9$	$1/3$	1	3	9	27	81

Example: Evaluate the following without a calculator

a) $10^0 = 1$ b) $(-8)^0 = 1$ c) $(0.567)^0 = 1$ d) $\pi^0 = 1$

Example: Evaluate the following by using the law for negative exponents first. Your answer should be **expressed as a fraction.**

a) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

b) $6^{-3} = \frac{1}{6^3} = \frac{1}{216}$

c) $(-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{16}$

Negative Exponents for Fractional Bases (Yikes!)

Rule: $\left(\frac{a}{b}\right)^{-c} = \left(\frac{b}{a}\right)^{c}$

a) $\left(\frac{1}{5}\right)^{-3} = 5^3$

$= 125$

b) $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2$

$= 4^2/3^2$

$= 16/9$